United States Naval Postgraduate School

THESIS

THE EFFECT OF SENSOR ACCURACY ON NAVAL GUNFIRE SUPPORT MISSION EFFECTIVENESS

by

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September 1970

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Naval Gunfire Support Mission Effectiveness

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the
NAVAL POSTGRADUATE SCHOOL
September 1970

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ABSTRACT

This thesis investigates the effects of errors in two shipboard sensors, the gyrocompass system and the peloruses, on a ship's mission effectiveness. The missions considered were a series of specially constrained shore bombardment missions. Various gyrocompass errors were investigated against area targets of varying radii.

The ultimate benefit which will hopefully be realized is that force commanders will be provided with a means to quantitatively evaluate the inherent capabilities of the various ships under their commands in assigning ships to specific missions.

In addition, a tactical innovation is suggested which could improve naval shore bombardment capabilities by partially countering the deleterious effect of ship's gyro error in indirect fire missions where spotting is not available.

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TABLE OF SYMBOLS AND ABBREVIATIONS

ATL	"Actual Target Line," line joining actual ship's position with actual target's position.
h	Actual ship's heading in ATL-cartesian coordinate system.
α	Actual bearing to reference (indicated by subscript) in ATL-cartesian coordinate system
e	Error in ATL-cartesian coordinate system.
$\wedge$	Superscript indicating quantity is measured clockwise in the geographical-polar coordinate system.
/	Superscript indicating quantity contains gyro or pelorus error (exceptions are specially denoted); description of the quantity is preceded by the word "Apparent" vice "Actual".
g	Gyro, subscript.
p	Pelorus, subscript.
T	Target, subscript
A	Aimpoint, when used as subscript; or maximum excursion from settled gyro error.
H	Projectile point of input, subscript.
ϕ	Angular displacement between true north in geographical-polar coordinate system and x-axis in ATL-cartesian coordinate system.
δ	0 or 1.
Δ	0 or 1; or incremental change.
μ_g	Settled gyro error.
$N(\mu,\sigma)$	Normal distribution with mean μ , and standard deviation σ.
P_o	Firing ship's position.
O	Firing ship, subscript.
B_r	Angle to target relative to ship's head, measured clockwise.
R_i	Navigational reference location of i^{th} reference.

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t_i	Tangent of $/_i$.
θ'_T	Angle from apparent ship's position to actual target in ATL-cartesian coordinate system
γ	Angle between x-axis and LOS
LOS	"Line of sight" horizontal line along which gun is pointed.
$\perp$LOS	Perpendicular to LOS.
Y_T	Distance from ship to target.
Y_b	Distance from ship to shore line.
d_T	Distance from ship to target.
d'_T	Distance from apparent ship's position to target.
d_e	Amount of error in computing range to target.
d_A	Distance between target and point of aim.
ξ	Displacement of projectile impact point from aimpoint, LOS component.
η	Displacement of projectile input point from aimpoint, $\perp$LOS component.
TACSIT	"Tactical Situation", a shore bombardment mission scenario.

I. INTRODUCTION

A. BACKGROUND

The United States Navy periodically sends combatant ships through Fleet Operational Readiness Accuracy Check Site (FORACS) calibration ranges to determine the individual ship's radar, sonar, gyrocompass, and pelorus errors. In the event that the ship passes in all categories, the measured errors are recorded but there is no relative measure which compares one ship's capabilities against those of another or enables such a comparison to be made, nor is there any method for quantitatively assessing the ship's inherent capabilities to perform an operational mission or task.

B. OBJECTIVES

It was the purpose of this paper to investigate the effects of gyrocompasses and pelorus errors upon a ship's capability to perform a specific mission. The mission chosen was a naval gunfire support mission since this involves both gyrocompass and pelorus in determining the ship's position, and the gyrocompass heading as an input to the gunfire control problem in laying the guns.

Naval gunfire, as it is normally practiced, relies heavily on observers to spot the fall of shot and send those spots to the ship so that adjustments to the fire control solution can be applied. An alternate tactical situation would be pre-invasion area fire where no spotting is available, and the purpose of the mission is to saturate the area with gunfire; even in this case rough corrections could be applied by visually spotting fall of shot from the ship.

In order to make this problem interesting, and the investigation meaningful, the scenario was an area fire mission, with the center of the area as the specific point of aim. The firing was conducted in its entirety without the benefit of any observation of fall of shot. In addition, the navigational position of the ship was fixed by obtaining two lines of position from references ashore. The charted position of the target and the navigational references were assumed to be accurate.

It is important to keep in mind that this investigation was intended to examine the degradation of a ship's mission effectiveness as a result of erroneous information received from two of the ship's sensors, the gyrocompass and the pelorus. It was not an investigation intending to correct all naval gunfire support problems.

II. DESCRIPTION OF THE GYROCOMPASS, PELORUS, AND MISSION

A description of the situation modeled breaks down into three broad categories:

a. A description of the gyrocompass, its behavior, its importance to the mission, and accompanying assumptions.

b. A description of the pelorus, its importance to the mission, and accompanying assumptions.

c. A description of the mission, and accompanying assumptions.

A. THE GYROCOMPASS

The heart of the gyrocompass is a gyroscope rotor. A rotor, if mounted with three degrees of freedom, and rotating at a sufficiently high constant speed, will maintain itself in alignment with a distant point in space. When the rotor is properly mounted within a framework of weights, it can be caused to align its axis parallel to the local horizontal plane, and point its axis toward true north. By this process a gyroscope is converted to a gyrocompass which will seek the local meridian and true north regardless of the earth's rotation or movement of the vehicle in which the gyrocompass is mounted.

In seeking the local meridian and true north, the gyro axis traces out a shallow ellipse. This hunting can be damped out so that the axis traces out an elliptical spiral. While the axis is seeking true north, any measurement from the gyro will be in error by the angular difference between the axis and an actual line pointing toward true north. A time plot of an undamped gyro's error would be a sine curve with a period of

84.4 minutes (time)[1]; a damped gyro's error would plot as an oscillatory curve of decreasing amplitude. The period of the first full oscillation would be approximately 87 to 89 minutes.

Oscillating errors in the gyrocompass are induced by a variety of causes such as the motion of the transporting ship on constant course and speed, acceleration of the ship, or rolling and pitching due to wave action.

An easily read heuristic explanation of the gyrocompass is provided in chapter 10 of reference 1. A detailed analytical explanation of the gyrocompass and its behavior can be found in chapter 10 of reference 2.

The gyro error is important because the ship's heading (including the error) is an input to the fire control solution, also to the pelorus which is used in determining the ship's position.

For purposes of this study it was assumed that the gyro error could be approximated by a sine curve. It was reasoned that shortly before the commencement of a gunfire support mission and during the first hour and a half of the mission (one period) some influence would cause the gyro to oscillate. It was further assumed that oscillations would not be additive or cancelling in nature, but that a second influence would serve to perpetuate an existing oscillatory motion, rather than allowing the motion to damp out.

B. THE PELORUS

The pelorus is essentially a remote gyrocompass repeater, mounted in a fixed stand located on the wings of the bridge (or other convenient

[1]The value 84.4 was derived analytically, and is explained in reference 2, pages 254-257.

location). Mounted on the pelorus is a rotatable circle with a telescope through which an observer can visually measure lines of bearing to references ashore, with respect to true north.

The observed bearing to a reference consists of the true bearing plus the gyro error at the instant the bearing is measured, plus a random effect which is a function of the particular repeater and observer. The random effect was assumed to be normally distributed with mean zero. It was assumed that the peloruses were mounted in perfect alignment with the ship's centerline so that there would be no fixed error inherent in the instruments.

C. THE MISSION

The mission, as briefly described in the introduction was one wherein a combatant ship with 5"/38 gun mounts, operating off a hostile shore, takes under fire a target of known geographical position, without the benefit of any means of observing fall of shot. The ship's position is determined by two bearings to fixed geographical references (of known position), the angle between the two lines of bearing was approximately 90°.

The target was an area target at sea level with the center known and the radius a variable. Several values were considered to suit the objectives of the reader. In this study the radius of the area target was varied from 0 to 1000 yards.

In order to facilitate modeling the situation described above, certain assumptions were made. The first assumption was that no attempt would be made to correct or compensate for the ship's gyrocompass error; after all, this was the effect under investigation.

Secondly, the effect of ship's motion through the water was assumed to be nil. To compensate for this assumption the navigational position of the ship was determined concurrently with each firing. Ignoring ship's motion initially seemed very contrived, but it was reasoned that the effects of wind and current could easily have a greater effect upon dislocating the ship from its intended track than the gyrocompass error. Moreover, the amount of displacement from the intended track because of gyro error at any time during the interval between determinations of position would be very small, since position would normally be determined or verified frequently. Under these conditions the assumption was considered acceptable. To amplify this point, consider a ship traveling 2° to the right or left of its intended track, with a speed of advance (over the ground) of 6 knots (nautical miles per hour). This ship would only be displaced 21 yards from its intended position at the end of three minutes (a reasonable interval between determinations of position). Furthermore, since ship's speed, effects of wind and current, and time interval between determination of position are all assumed to be random, the determination of position with each firing served to compensate for all of these effects.

Thirdly, it was assumed that negligible battery alignment error existed between the guns and the computer. In order to account for effects of wind, temperature, variations in propellant weight and temperature, gun wear, and others; a single ballistic random error was used as an input to provide some measure of dispersion about the point of aim. The ballistic errors referred to were similar to those contained in references 3, 4 and 5.

A fourth assumption required was that the firing ship would not conduct the firing mission while at anchor, but rather at slow speed. This assumption may seem contradictory to the second, but it was important that the ship not be restricted to one known geographical position throughout the firing. The second assumption, although it discounted the effects of wind and current, does require that the ship's position be determined concurrently with each shot. By restricting the ship to slow speeds, it was tacitly assumed that any relative bearing dependent error in the peloruses would be voided.

Lastly, it was assumed that the firing ship would steam roughly parallel to a long straight shore line while conducting the firing mission with the target approximately on the beam. This assumption was a realistic one since in steaming parallel to the shore there would be only minor changes in range to the target. This is normally desired in shore bombardment missions since some ballistic corrections which are manual inputs to the fire control solution are range dependent. An additional consideration is that the dispersion of the projectiles is a function of range, and if the range varied appreciably, then data on projectile hits taken during one time interval might not be commensurable with data obtained during a different time interval.

Prevalent characteristics of ship's superstructures are such that in steaming parallel to the shore, peloruses on the disengaged side of the ship would be effectively blocked. It was reasonable to infer therefore that only one pelorus would be used in sighting navigational references, with an insignificant time delay between sightings.

In practice, the CIC (Combat Information Center) personnel plot the ship's position and transmit range and bearing of the target to the plotting room where it is introduced into the fire control computer (the effects of wind and current are included). As soon as the computer begins to generate a consistent and apparently correct solution, the gunfire mission commences. All of this takes place within a relatively short time span and the effect of oscillation in the gyrocompass, due to changes of the ship's course and speed in preparation for the gunfire mission, and other causes, may not be fully apparent for several minutes.

III. MODELING THE SITUATION

A. COORDINATE SYSTEMS AND CONVENTIONS

The first step in modeling the gyro error problem was to define the coordinate systems being used, and establish conventions for using them.

In discussing gyros or peloruses, and the associated errors, the usual convention is to refer to a polar coordinate system with a true north reference, incremented in degrees increasing from 0 to 360 in a clockwise direction.

A gyro error is that angular measure between the ship's true heading (with reference to true north) and its apparent heading. If the error is such that the ship's heading is actually to the right of its apparent heading, this is commonly called an easterly error. Conversely, if the error is such that the ship's actual heading is to the left of its apparent heading, this is called a westerly error, see Figure 1 (a).

Let $\hat{h}$ be the actual heading of the ship measured clockwise from true north and let $\hat{h}'$ be the apparent heading, also measured from true north. By letting $\hat{e}_g$ represent the gyro error, it is clear that:

$$\hat{h} = \hat{h}' + \hat{e}_g$$

if the convention is concurrently established that:

$\hat{e}_g > 0$ represents an easterly error, and

$\hat{e}_g < 0$ represents a westerly error.

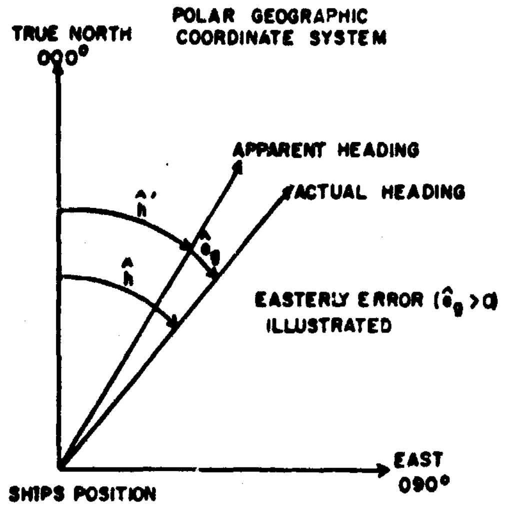

FIG. 1(a)

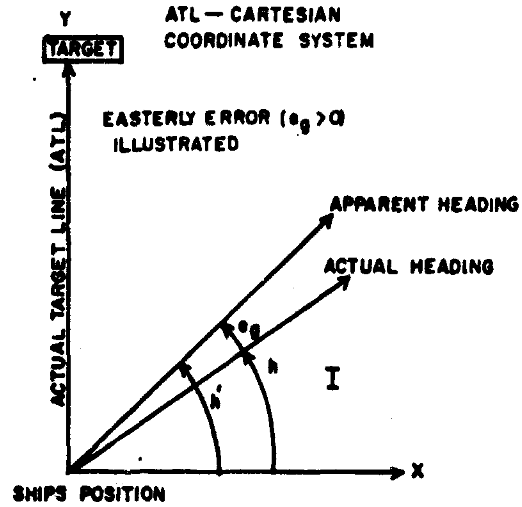

FIG. 1(b)

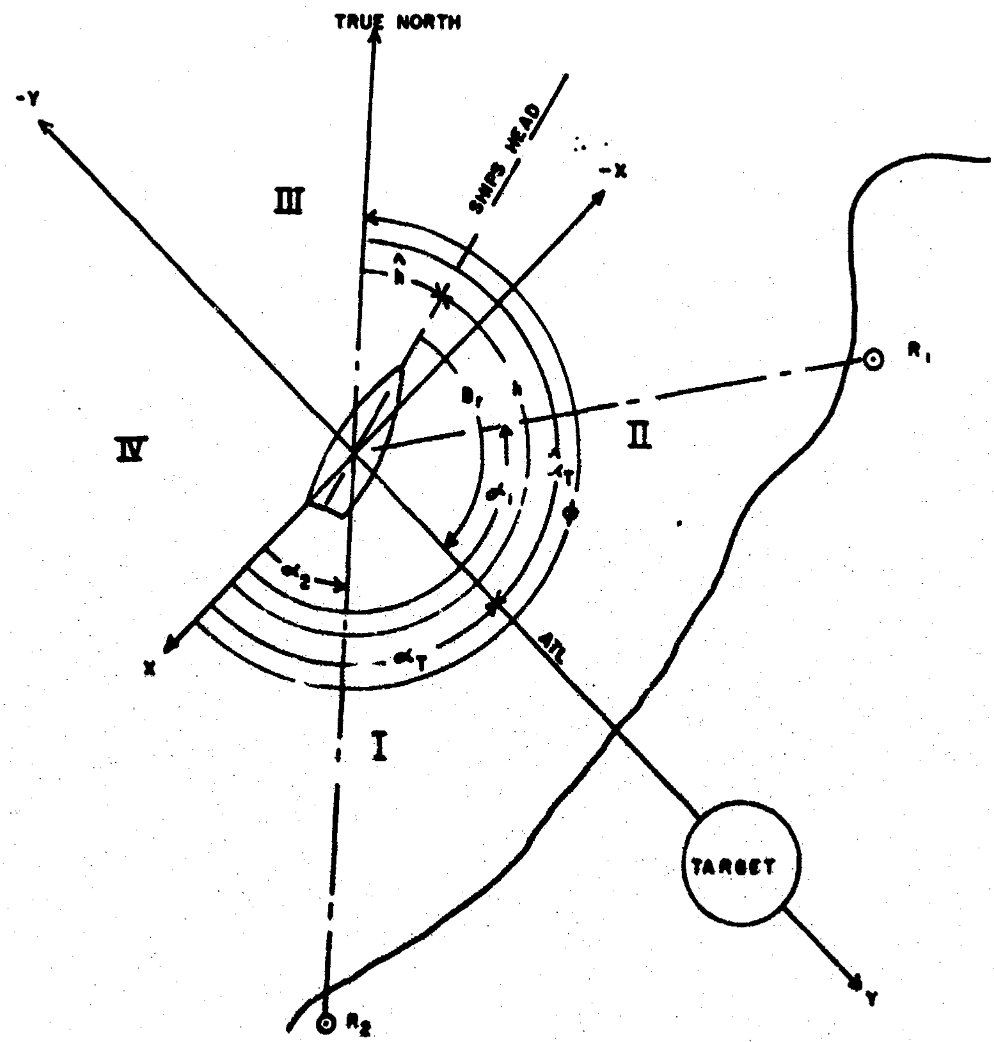

FIG. 1(c) RELATIONSHIP BETWEEN COORDINATE SYSTEMS

For the purposes of constructing a model of the gunfire support mission, this geographic polar coordinate system would become most unwieldy. Therefore, the problem was modeled using the cartesian coordinate system with origin at the ship's actual position and the positive y-axis formed by a line from the ship's true position to the true position of the target's center. This line was called the actual target line and abbreviated ATL. The positive x-axis was 90 degrees clockwise from the positive y-axis, intersecting the y-axis at the origin, the ship's actual position.

In measuring angular position within the cartesian coordinate system the conventional practice of measuring in degree increments from 0 to 360 counter-clockwise from the positive x-axis was utilized.

Since the gyro errors were of such fundamental importance in both coordinate systems, it was convenient to maintain the already established conventions; therefore, in the ATL referenced cartesian coordinate system:

$e_g > 0$ represents an easterly error, and

$e_g < 0$ represents a westerly error.

It was immediately apparent that $\hat{e}_g = e_g$. It was also apparent that since the angular measurements were in opposite directions in the two coordinate systems that a new defining equation would be needed. Letting h represent the actual ship's heading and h' the apparent ship's heading in the cartesian coordinate system,

$$h' = h+e_g,$$

see Figure 1 (b).

The superscript $\wedge$ was used to distinguish variables in the geographical polar coordinate system, from variables in the cartesian coordinate system, which would not be so marked. Similarly, the $/$ superscript was used to distinguish an apparent bearing or heading from an actual bearing or heading.

The two coordinate systems were related through an angle $\emptyset$ which was measured counterclockwise from the X-axis of the cartesian coordinate system established by the ATL, to true north in the geographical polar coordinate system. The two coordinate systems were related by the equation:

$$h = (\emptyset - \hat{h}) + \delta \cdot 360°$$

$$\text{where} \quad \delta = \begin{cases} 0 & \text{if } (h + \hat{h}) < 360° \\ 1 & \text{otherwise,} \end{cases}$$

$$\text{and} \quad 0 \leq \emptyset < 360°$$

Letting α represent bearing angles in the cartesian coordinate system and $\hat{\alpha}$ represent bearing angles in the geographical polar coordinate system, the angle to the actual target in the cartesian coordinate system would be 90° by definition, or $\alpha_T = \pi/2$ radians. In the geographical polar coordinate system:

$$\hat{\alpha}_T = (\emptyset + \Delta \cdot 360°) - 90°$$

$$\text{where} \quad \Delta = \begin{cases} 1 & \text{if } \emptyset \quad 90°, \\ 0 & \text{otherwise.} \end{cases}$$

By substitution it is apparent that:

$$\hat{\alpha}_T + \alpha_T = \phi + \delta \cdot 360°$$

$$\text{where} \qquad \delta = \begin{cases} 0 \text{ if } (\alpha_T + \hat{\alpha}_T) < 360° \\ 1 \text{ otherwise.} \end{cases}$$

The ship's heading, a necessary input to the gunfire control solution, is transmitted to the fire control computer with the gyro error included,

$$\hat{h}' = \hat{h} - \hat{e}_g.$$

The angle to the target relative to the ship's heading, which is necessary for accurately laying the guns is, $B_r = \hat{\alpha}_T - \hat{h}$ (B_r measured clockwise from the ship's heading), but the value received by the computer is $B_r' = \hat{\alpha}_T - \hat{h}'$ or $B_r' = \hat{\alpha}_T - (\hat{h} - \hat{e}_g)$; $\hat{\alpha}_T$ is received accurately (presuming the ship's position is correctly determined) since the target's true position is obtained from a chart or map.

In the mathematical model of the problem the bearing of the target relative to the ship's heading became unimportant. It was assumed that the ship steams roughly parallel to the shore and it was tacitly assumed that the guns could be brought to bear on the target without obstruction by the ship's superstructure. By substitution it is easily shown that:

$$\begin{aligned} B_r' &= \hat{\alpha}_T - (\hat{h} - \hat{e}_g) \\ &= [(\phi + \Delta \cdot 360) - 90] - [\phi - h + \delta \cdot 360] + \hat{e}_g, \end{aligned}$$

and since $e_g = \hat{e}_g$,

$$B_r' = \Delta \cdot 360 - 90 + h - \delta \cdot 360 + e_g$$

or, since $\alpha_T = 90°$

$$B_r' = \Delta \cdot 360 - \delta \cdot 360 + h - \alpha_T + e_g .$$

Therefore, since B_r', which is used in the solution of the fire control problem, can be expressed as a function of variables of either coordinate system, and the same gyro error is included in both expressions, the variables of the cartesian coordinate system alone can be used and the value of target bearing relative to the ship's head was omitted.

A correct indirect fire control solution is as dependent upon knowledge of the ship's true position as it is upon knowledge of the target's true position. The ship's position is obtained by plotting the intersection of lines of bearing from fixed references whose positions are known. The lines of position as used, contain both the time dependent gyro error and a pelorus error e_p, which has been assumed to be normally distributed. The apparent bearing to reference i in the geographical polar coordinate system would be:

$$\hat{\alpha}_i' = \alpha_i - (\hat{e}_{g_i} + \hat{e}_{p_i}),$$

whereas in the ATL referenced cartesian coordinate system the apparent bearing is:

$$\alpha_i' = \alpha_i + (e_{g_i} + e_{p_i}) .$$

Again $e_g = \hat{e}_g$, similarly $e_p = \hat{e}_p$.

Transposing from one coordinate system to the other by use of the angle ϕ can be accomplished as previously demonstrated for bearing to the target. The interrelationship of the variables described is depicted in Figure 1 (c).

B. THE ANALYTICAL GYRO-PELORUS MODEL

In the gyrocompass - pelorus model, using the ATL referenced cartesian coordinate system, the origin was the ship's actual position $P_0 = (X_0, Y_0) = (0, 0)$, the target's actual position is along the ATL and was described as $(X_T, Y_T) = (0, Y_T)$. The two navigational references were described $R_1 = (X_1, Y_1)$ and $R_2 = (X_2, Y_2)$ respectively, The apparent angles to the two references were:

$$\alpha'_1 = \alpha_1 + (e_{g1} + e_{p1})$$

and

$$\alpha'_2 = \alpha_2 + (e_{g2} + e_{p2}) \text{ respectively.}$$

The erroneous pelorus bearings resulted in an incorrectly plotted ship's position:

$$P'_0 = (X'_0, Y'_0) .$$

The point P'_0 was solved for in the simultaneous equations:

$$Y'_0 = (X'_0 - X_2) \tan \alpha'_2 + Y_2$$
$$Y'_0 = (X'_0 - X_1) \tan \alpha'_1 + Y_1 .$$

The solution of these equations yielded:

$$X'_0 = [(X_1 t'_1 - X_2 t'_2) - (Y_1 - Y_2)] \,/\, (t'_1 - t'_2)$$
$$Y'_0 = \Big([(X_1 t'_1 - X_2 t'_2) - (Y_1 - Y_2)] \,/\, (t'_1 - t'_2) \Big) t'_1 - X_1 t'_1 + Y_1 .$$

where $t'_i = \tan \alpha'_i, \quad i = 1,2.$

This geometry is depicted in Figure 2.

Using the erroneous information which resulted in the ship's position being plotted at P'_0 , the calculated distance to the target was:

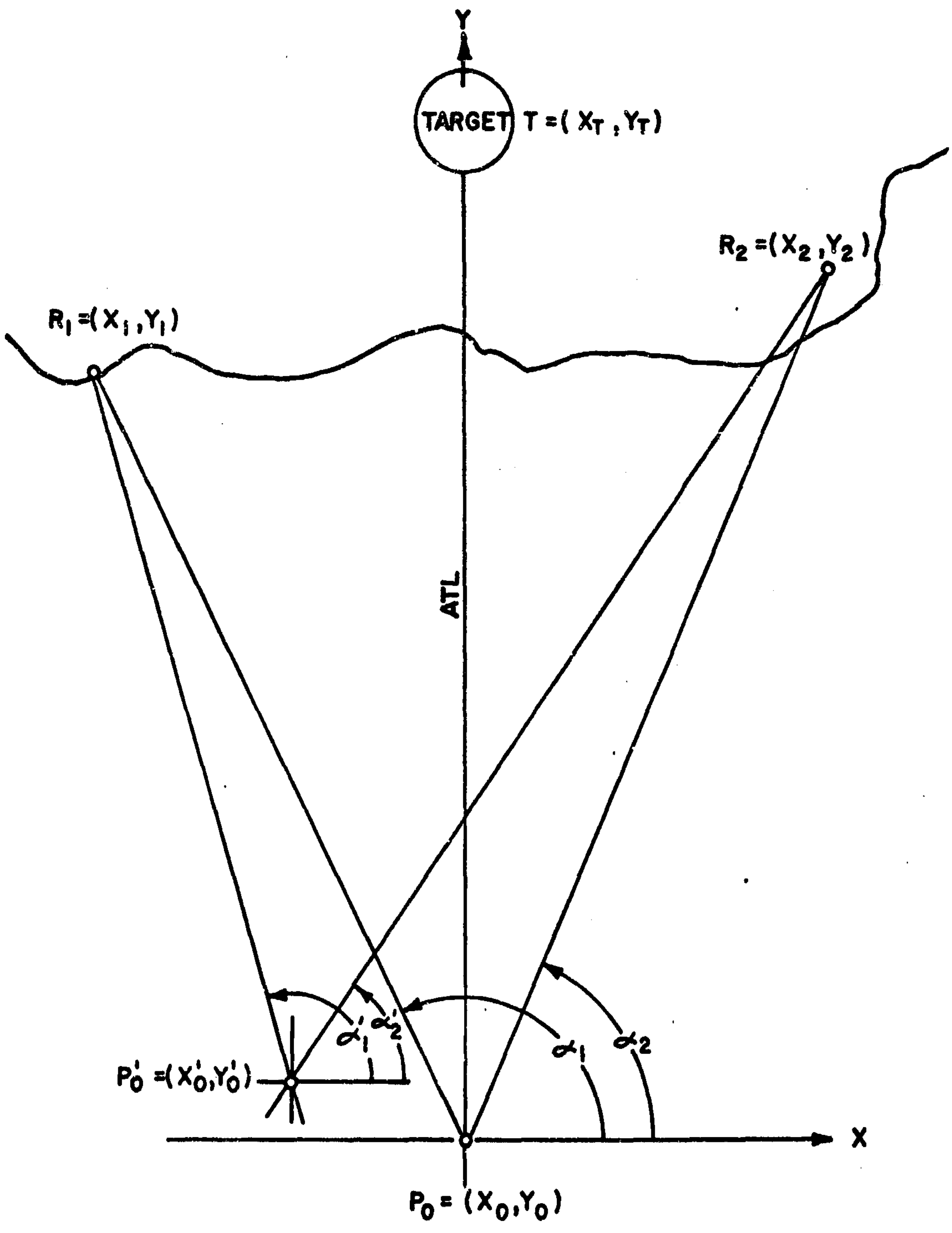

FIG. 2 GEOMETRY FOR THE GYRO-PELORUS MODEL

$$d'_T = \sqrt{(X'_0)^2 + (Y_T - Y'_0)^2} \quad .$$

This value was used in the fire control solution rather than the correct value:

$$d_T = Y_T \; .$$

The apparent angle to the target due solely to incorrectly plotting the ship's position was:

$$\theta'_T = \tan^{-1}\left[\frac{Y_T - Y'_0}{-X'_0}\right],$$

which is analogous to α_T (use of α_T depended upon position having been plotted correctly). When combined with the angular error due to gyro, introduced through ship's heading, the angle that the ship would fire on, as a result of erroneous position and heading information would be:

$$\gamma = \theta'_T - e_g \; .$$

The actual point of aim was then described by:

$$X_A = d'_T \cdot \cos\gamma \quad .$$
$$Y_A = d'_T \cdot \sin\gamma \quad .$$

The relationship between the aimpoint and the target's actual position is depicted in Figure 3.

If there were no ballistic errors, no errors in battery alignment, no errors caused by meteorological effects and no error in the transmission of information within the shipboard fire control circuitry and

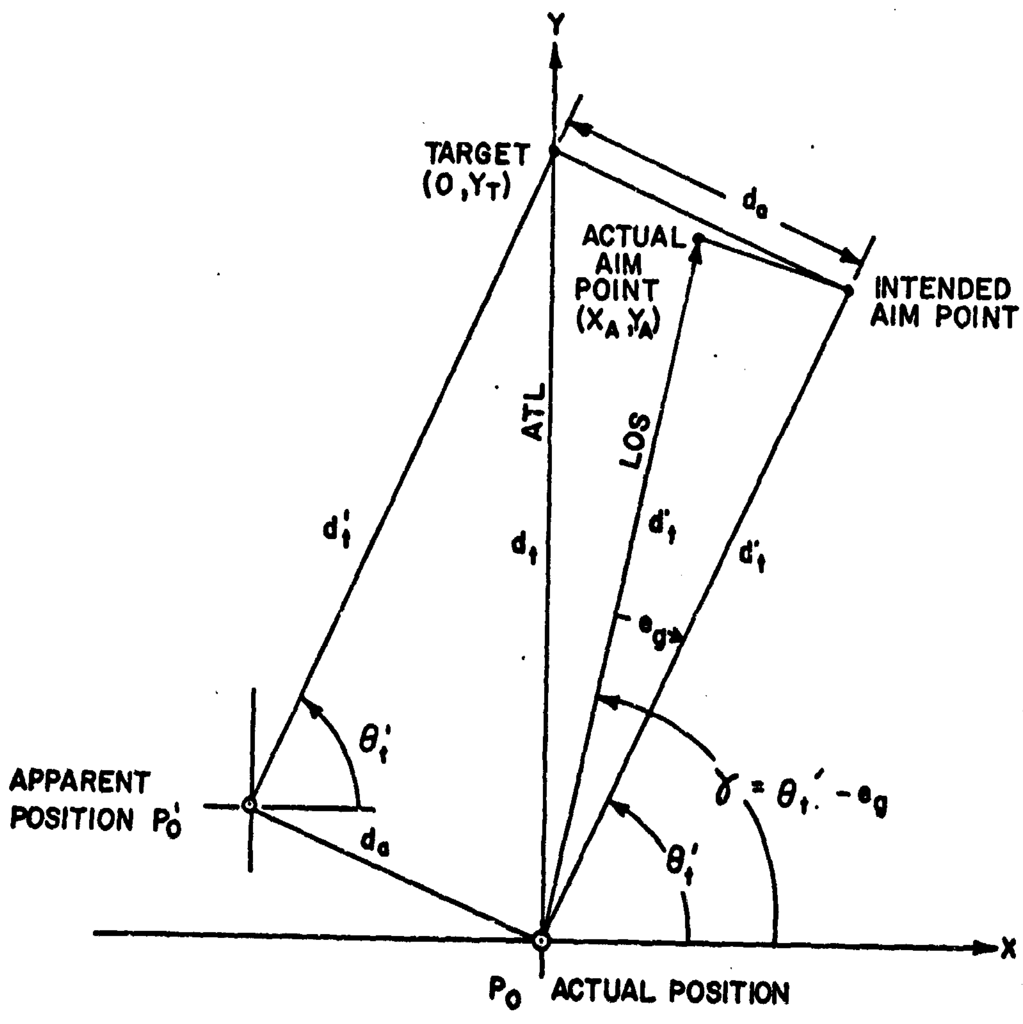

FIG.3 RELATIONSHIP BETWEEN THE AIM POINT AND THE ACTUAL TARGET POSITION

mechanisms; then each round fired would hit the point of aim as described by (X_A, Y_A), and the miss distance would be:

$$d_A = \sqrt{[X_A^2 + (Y_A - Y_T)^2]} \quad .$$

As explained previously, all random errors other than those within the gyroscope and pelorus will be lumped into a single ballistic error.

The ballistic errors are assumed to adhere to a bivariate normal distribution with complete independence between in-LOS and across-LOS components[2], and means in and across the LOS both equal to zero. The mean point of impact for the projectiles would therefore be located at the aimpoint (X_A, Y_A). An excellent explanation of ballistic errors is contained in reference 6.

[2]Spin stabilized projectiles will drift across the LOS. The amount of drift is a function of range, therefore the assumption of independence was not strictly valid; however the relative effect of drift is small and this assumption was considered reasonable.

IV. MODEL APPLICATIONS

For the gunfire support mission considered, the measure of effectiveness utilized was the probability that any round hits within a prescribed radius from the target center.

Since the aimpoint is a complicated function of errors e_g and e_p, its distribution would be difficult to determine analytically. Accordingly, the aimpoint coordinates were obtained numerically by drawing the errors e_g and e_p from their distributions and applying them in the model. For each aimpoint so obtained a ballistic error was drawn and applied to the aimpoint to yield a hit point. In this way a cumulative distribution of miss distances was constructed for expanding radii about the target center.

The data required to use the model are:

a. Gyro error characteristics (settled error, and maximum excursion from the settled error).

b. Pelorus error characteristics (mean and variance).

c. Ballistic error characteristics (mean and variance both along and perpendicular to the gun LOS).

The ship's gyro error was obtained by drawing uniformly from a sine curve having as its mean the gyro's settled error, μ_g, and as its amplitude the gyro's maximum excursion from the mean settled error, A; see Figure 4. Both values can be obtained from FORACS data for any ship which has been through the range; reference 7 provides examples.

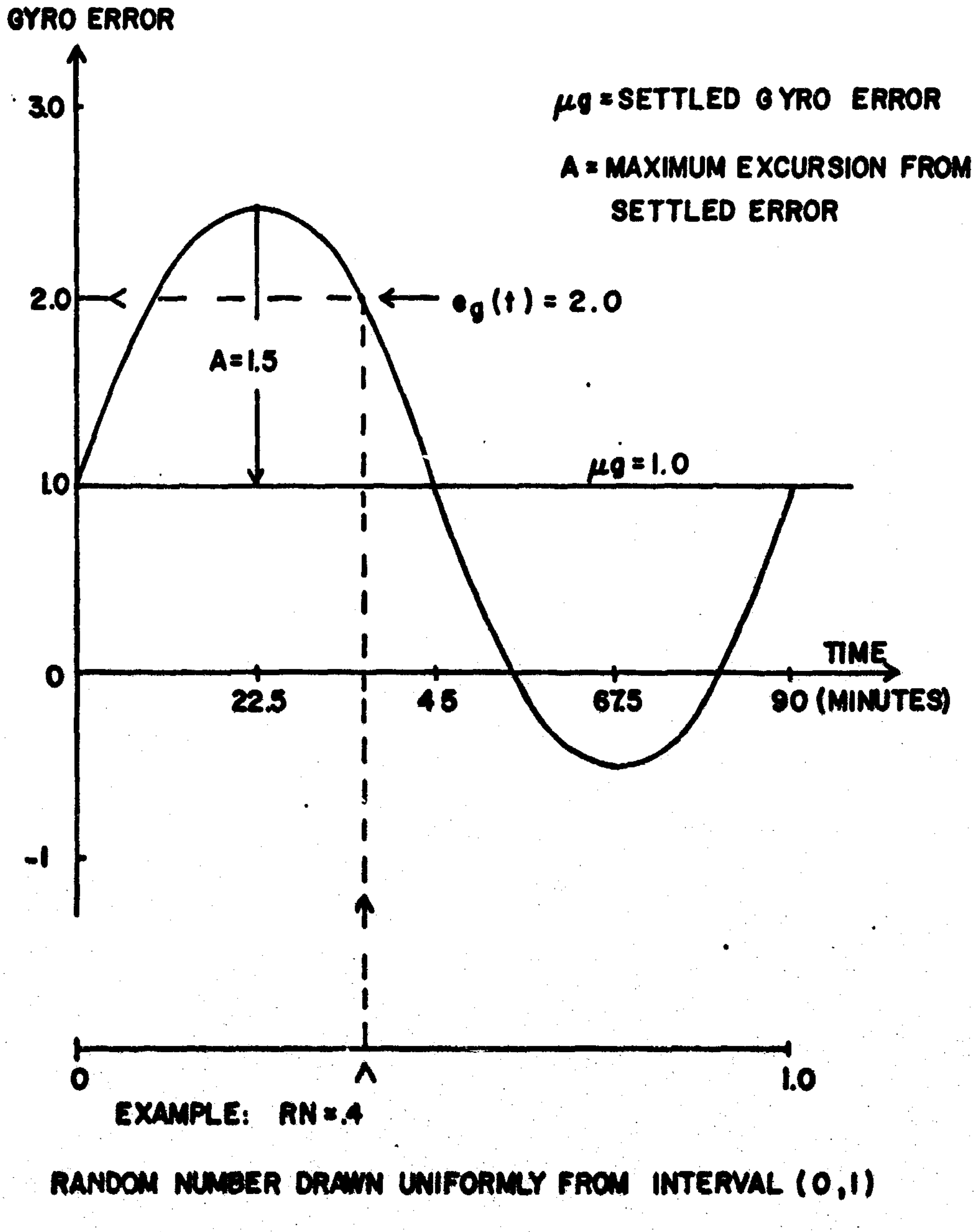

FIG. 4 METHOD OF DRAWING GYRO ERROR

For illustrative purposes, a range of means of two degrees was utilized, with μ_g varying in half degree increments; i.e.:

$$-1.0 \leq \mu_g \leq 1.0 ,$$
$$\Delta \mu_g = .5 .$$

The maximum excursion about the settled error was also varied in half degree increments; a range of amplitude variation of one degree was considered,

$$.5 \leq A \leq 1.5 ,$$
$$\Delta A = .5 .$$

The pelorus errors, exclusive of gyro error were assumed to be normally distributed, having the actual bearing as a mean and a relatively small variance. For purposes of this study the pelorus error was assumed to have a mean of zero, and a variance of .3 degrees throughout. In this study each error was obtained by drawing uniformly from the normal distribution N(0, .3). The same pelorus error is not applied to both lines of bearing for any given shot.

The ballistic error, as noted previously, comes from a bivariate normal distribution, with the mean errors both along, and perpendicular to, the actual line of sight of the gun being zero. Ballistic error data can be obtained from references 3-5. For purposes of illustration, the standard deviations along the LOS, σ_{LOS}, were obtained from reference 3 and linearly extrapolated where necessary. A value of three mils[3] was arbitrarily chosen for the standard deviation across the LOS, $\sigma_{\perp LOS}$.

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[3]One mil is an error of one yard per one thousand yards of range.

The i^{th} hitpoint was described by (ξ_i, η_i) where $(X_A, Y_A) = (0,0)$, and η was an extension of the line joining (X_0, Y_0) and (X_A, Y_A), see Figure 5. The two coordinates were obtained by drawing uniformly from the normal distributions:

$$\xi \sim N(0, \sigma_{\perp LOS})$$

and $$\eta \sim N(0, \sigma_{LOS})$$ respectively.

In both distributions σ is a function of range to the target, type of gun, and type of powder and projectile.

Once the hitpoint was obtained in the gun's LOS coordinate system (ξ, η) it was transposed into the coordinate system determined by the ATL to the target by rotating (ξ, η) through $(\gamma - 90)$ degrees and translating. This coordinate system would then coincide with the coordinate system determined by the ship and target actual positions. This was done using the relationships:

$$X_H = \xi \cos(\gamma - \pi/2) - \eta \sin(\gamma - \pi/2) + X_A$$

$$Y_H = \xi \sin(\gamma - \pi/2) + \eta \cos(\gamma - \pi/2) + Y_A$$

where γ is measured in radians.

With (X_H, Y_H) determined, the actual miss distance to the target was determined as follows:

$$d_H = \sqrt{[X_H^2 + (Y_H - Y_T)^2]} \quad .$$

By repeated replications, a distribution of hits was constructed. By holding either A or μ_g constant, and increasing the other in discrete increments, a family of curves was obtained. This family

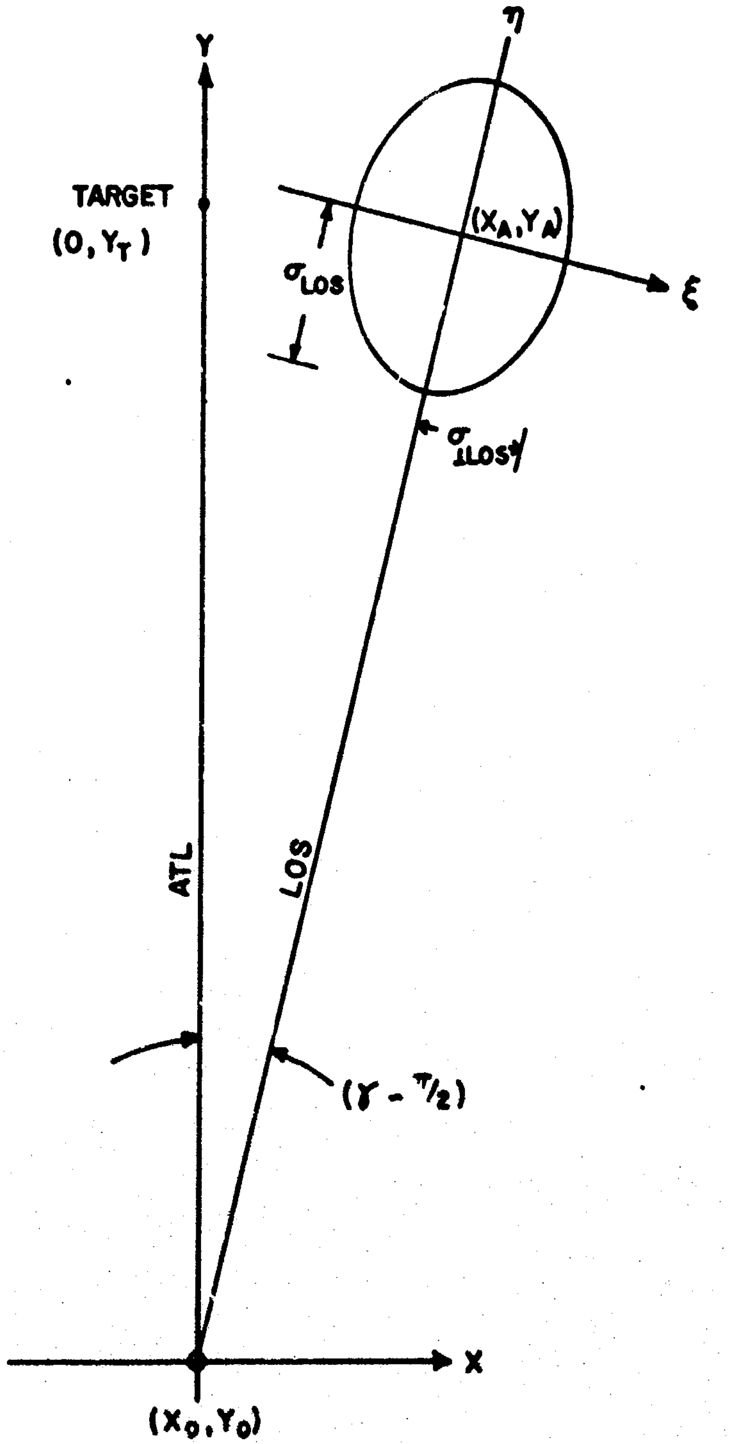


FIG.5 RELATIONSHIP BETWEEN LOS COORDINATE SYSTEM AND ATL COORDINATE SYSTEM

describes mission effectiveness, subject to the constraint imposed by the variable held fixed.

The model was employed in a variety of tactical situations, where range from ship to target, Y_T, and range from ship to shore line, Y_b, were each increased in increments of 3000 yards.

The tactical situations are described in Figure 6. The numerals in the upper left corner of each box in Figure 6 will be used hereafter for identifying purposes. Those tactical situations denoted with an asterisk are those which are considered to be out of the range of 5"/38 standard projectiles. In these cases, 5"/38 rocket assisted projectiles (RAP) would be appropriate. Ballistic data on RAP is obtainable in reference 4. Data used in this study was extrapolated from reference 3.

Sixteen of the tactical situations (TACSIT's) were symmetric in construction, i.e., the line joining the two references were perpendicular to the ATL from ship to target, with the two navigational references located at angles of 45° on either side of the ATL. An additional test was conducted on TACSIT 6 with non-symmetric construction to determine the sensitivity of the model to variations in the location of the navigational references; this was TACSIT 17.

The model was subjected to one thousand replications for each combination of the parameters (Y_T, Y_b, μ_g, A). The random numbers generated were identical for each tactical situation combination.

SUMMARY OF TACTICAL SITUATIONS AND DISPERSION DATA

Y_T and σ_{LOS} Entries are in Yards

Distance from shoreline to target, $(Y_T - Y_b)$, yards

Distance from ship to shoreline (Y_b), yards	3000	6000	9000	12000
3000	1 Y_T = 6000 σ_{LOS} = 48	2 Y_T = 9000 σ_{LOS} = 49	3 Y_T = 12000 σ_{LOS} = 58	4 Y_T = 15000 σ_{LOS} = 74
6000	5 Y_T = 9000 σ_{LOS} = 49	6 Y_T = 12000 σ_{LOS} = 58	7 Y_T = 15000 σ_{LOS} = 74	8* Y_T = 18000 σ_{LOS} = 88
9000	9 Y_T = 12000 σ_{LOS} = 58	10 Y_T = 15000 σ_{LOS} = 74	11* Y_T = 18000 σ_{LOS} = 88	12* Y_T = 21000 σ_{LOS} = 103
12000	13 Y_T = 15000 σ_{LOS} = 74	14* Y_T = 18000 σ_{LOS} = 88	15* Y_T = 21000 σ_{LOS} = 103	16* Y_T = 24000 σ_{LOS} = 118

$\alpha_1 = 135°$, $\alpha_2 = 45°$ in TACSIT's 1-16

17 (A modification of TACSIT 6) $Y_T = 12000$, $Y_b = 6000$ $\sigma_{LOS} = 58$ $\alpha_1 = 120°$, $\alpha_2 = 30°$

$\sigma_{\perp LOS}$ = 3 mils in all cases

* Range dispersion data was linearly extrapolated

FIGURE 6. Summary of Tactical Situations and Dispersion Data

V. RESULTS

Before investigating the results in detail, it is important to mention that there was no appreciable difference noted in the symmetric cases (TACSIT's 1-16) between easterly and westerly error performance curves; accordingly, westerly settled errors ($\mu_g < 0$) will not be used since all results can be adequately explained by referring to easterly error situations.

The results of TACSIT 4 where $(Y_T, Y_b) = (15000, 3000)$, are presented in figures 7-12. Significant characteristics observed in these results are summarized below. These observations are characteristic of all symmetric cases in which the range from ship to shore line did not equal the range from shore line to target, i.e., $(Y_T - Y_b) \neq Y_b$.

1. Holding A or μ_g constant and increasing the value of the other, degrades overall performance, in that the radius from target center, within which all of the projectiles fell, was increased.

2. For small values of A, on the order of A = .5, performance was seriously degraded for small targets if $|\mu_g| \geqq A$.

3. For larger values of A; A = 1.0, 1.5; the performance curves were very nearly the same for small radius targets for all values of μ_g investigated. Note on Figure 8 that the performance against a target of radius 100 yards is virtually identical for the combinations $(|\mu_g|, A) = (0.0, 1.0)$ or $(0.5, 1.0)$.

4. In cases where μ_g is small ($|\mu_g| \leqq .5$), increasing the value of A degraded performance markedly for small targets.

5. In tactical situation 4, it appeared that satisfactory performance was not attained against a very small target (radius $\leq$ 10 yards) for any combination of ($|\mu_g|$, A).

Figures 13 through 21 depict families of performance curves for tactical situations 1, 6, 11, 16, 4, 7, 10 and 13 respectively. In each figure the curves for the parameters (μ_g, A) = (0, .5), (.5, 1.0), and (1.0, 1.5) are shown, as well as a separate curve obtained when μ_g = 0, A = 0, and the mean and standard deviation of pelorus errors were also zero ($\mu_p = \sigma_p = 0$). This last curve is one depicting the ballistic error alone.

It was interesting to note that in tactical situations 1, 6, 11, and 16 (Figures 13-16) only two curves were necessary, one representing the performance achieved for ballistic error alone and the other representing the performance achieved by <u>all</u> of the other (μ_g, A) combinations investigated. In these situations the shore line is equidistant between ship and target, $Y_b = (Y_T - Y_b)$.

Figures 17 through 20 show the performance curves for tactical situations 4, 7. 10, and 13. These situations represented those cases wherein the range from ship to target was constant, Y_T = 15000 yards, and the distance from ship to shore line, Y_b, was increased in increments of 3000 yards. A comparison of these figures revealed a marked degradation in performance for any (μ_g, A) combination as Y_b diverged from $(Y_T - Y_b)$ in either direction.

Figure 21 depicts the performance curves for (μ_g, A) = (0, .5), (.5, 1.0). and (1.0, 1.5) obtained from a ship in tactical situation 17 wherein the navigational references are located at unequal angles (30° and 60°) on either side of the ATL. The line joining the references is still perpendicular to the ATL. Figure 21 also depicts the performance curves shown in Figure 14 from tactical situation 6 (symmetric construction). The marked degradation of performance noted in TACSIT 17 over that noted in TACSIT 6 can only be attributed to the non-symmetry of the navigational reference points with respect to the ATL.

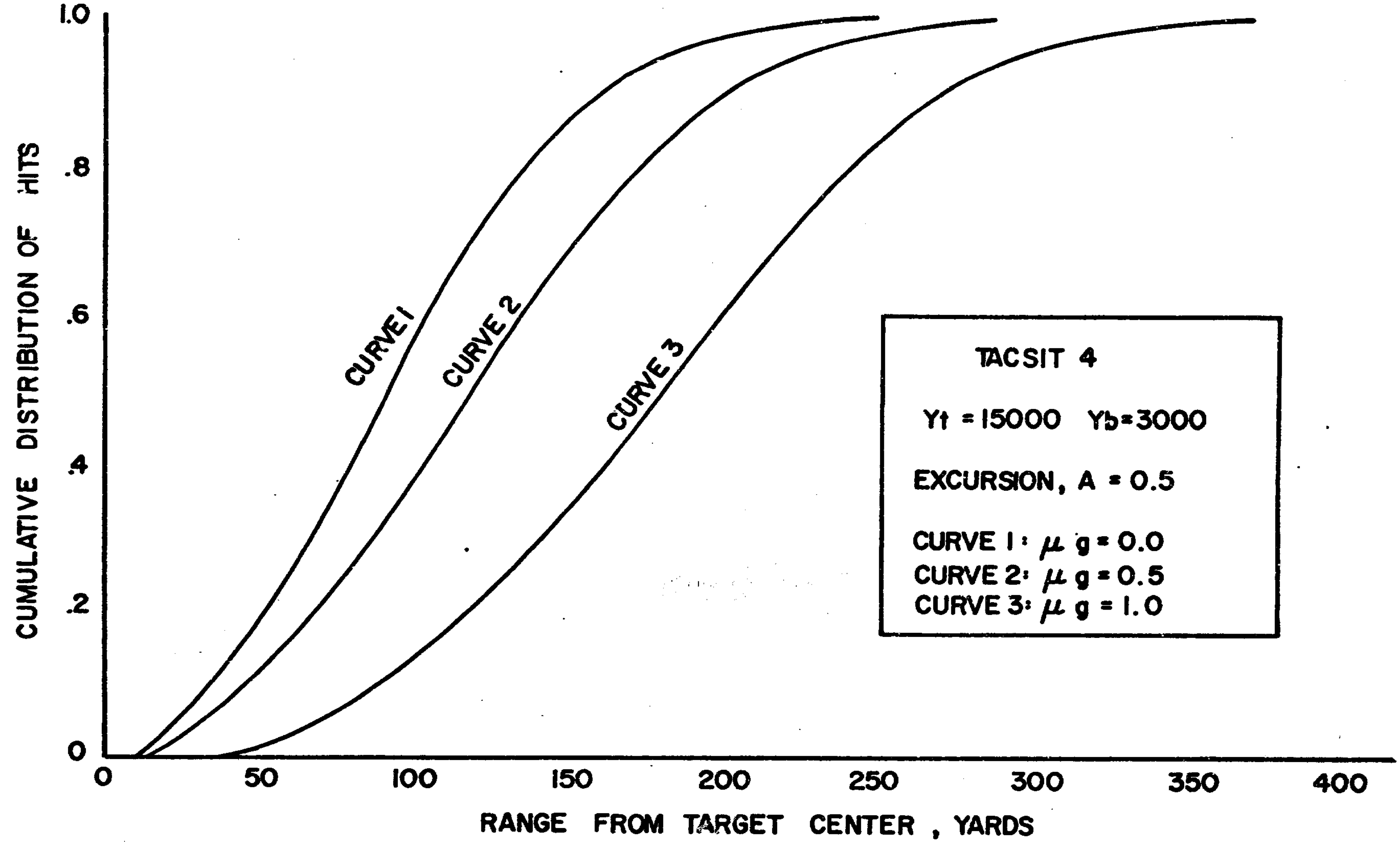
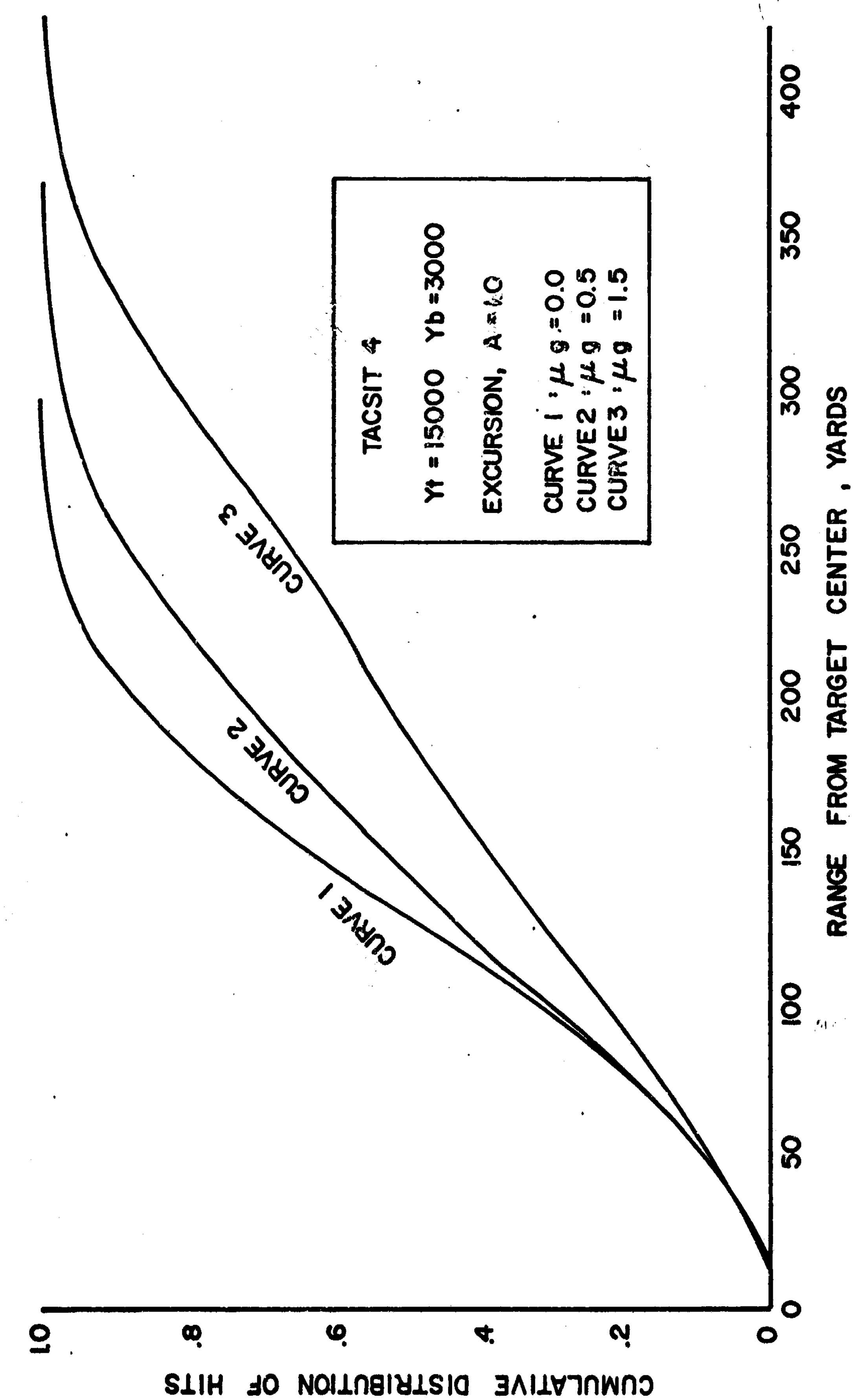

1.0
.8
.6
.4
.2
0
CUMULATIVE DISTRIBUTION OF HITS
CURVE 1
CURVE 2
CURVE 3
TACSIT 4
Yt = 15000 Yb = 3000
EXCURSION, A = 0.5
CURVE 1: μ g = 0.0
CURVE 2: μ g = 0.5
CURVE 3: μ g = 1.0
0
50
100
150
200
250
300
350
400
RANGE FROM TARGET CENTER, YARDS

FIG. 7



TACSIT 4
Yt = 15000 Yb = 3000
EXCURSION, A = 1.0
CURVE 1 : μg = 0.0
CURVE 2 : μg = 0.5
CURVE 3 : μg = 1.5
CURVE 1
CURVE 2
CURVE 3
CUMULATIVE DISTRIBUTION OF HITS
1.0
.8
.6
.4
.2
0
0
50
100
150
200
250
300
350
400
RANGE FROM TARGET CENTER , YARDS

FIG. 8

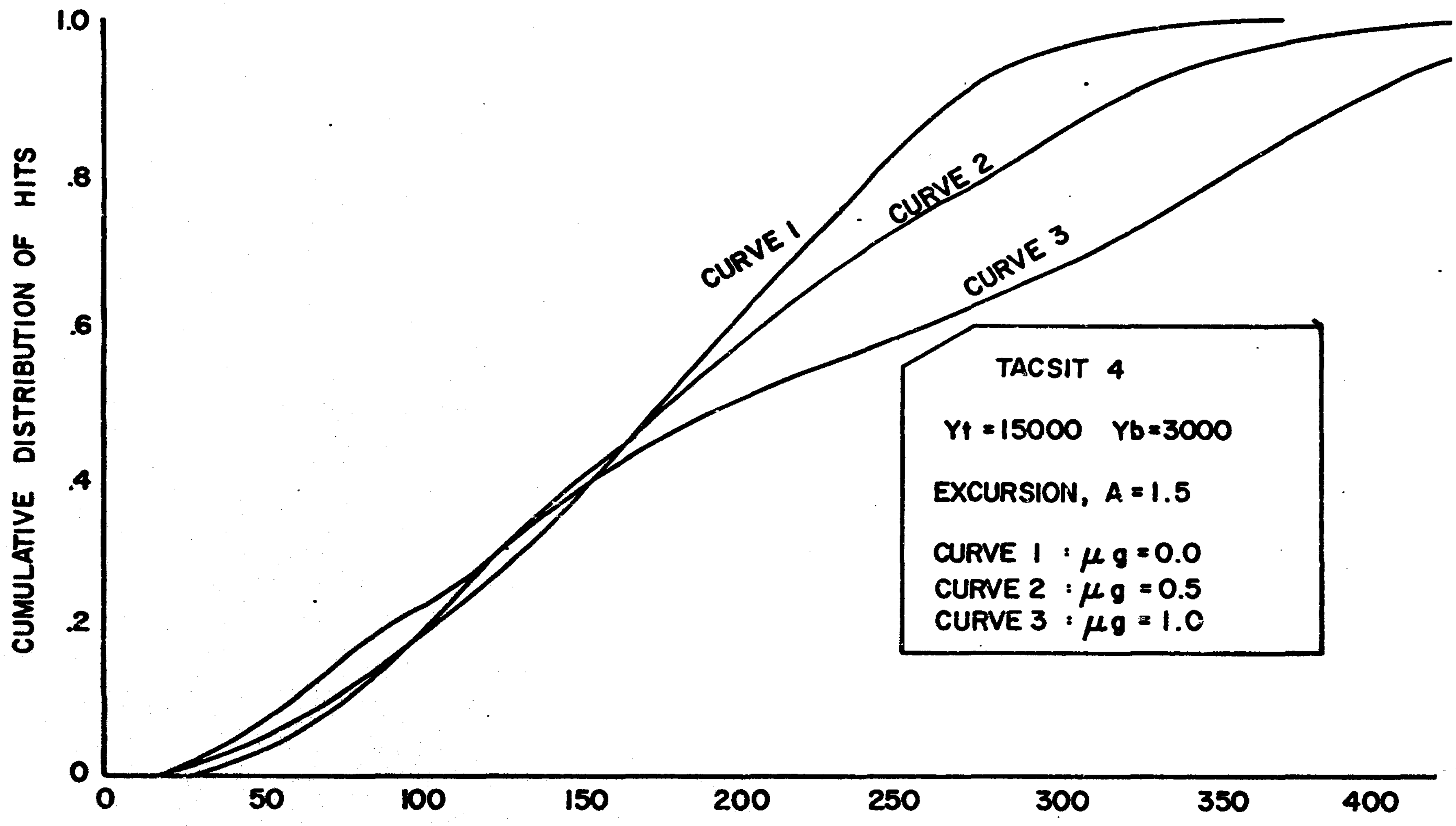
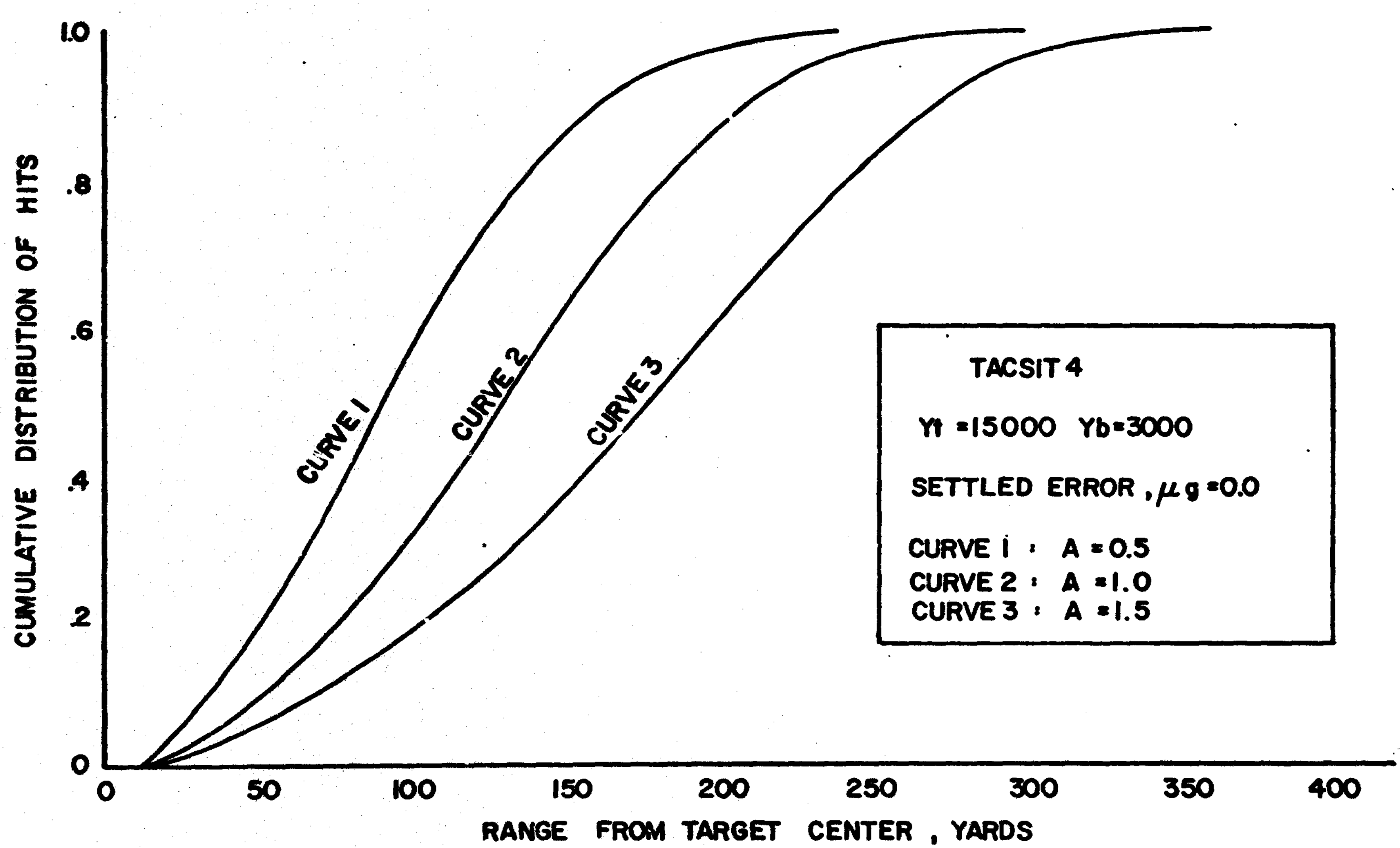
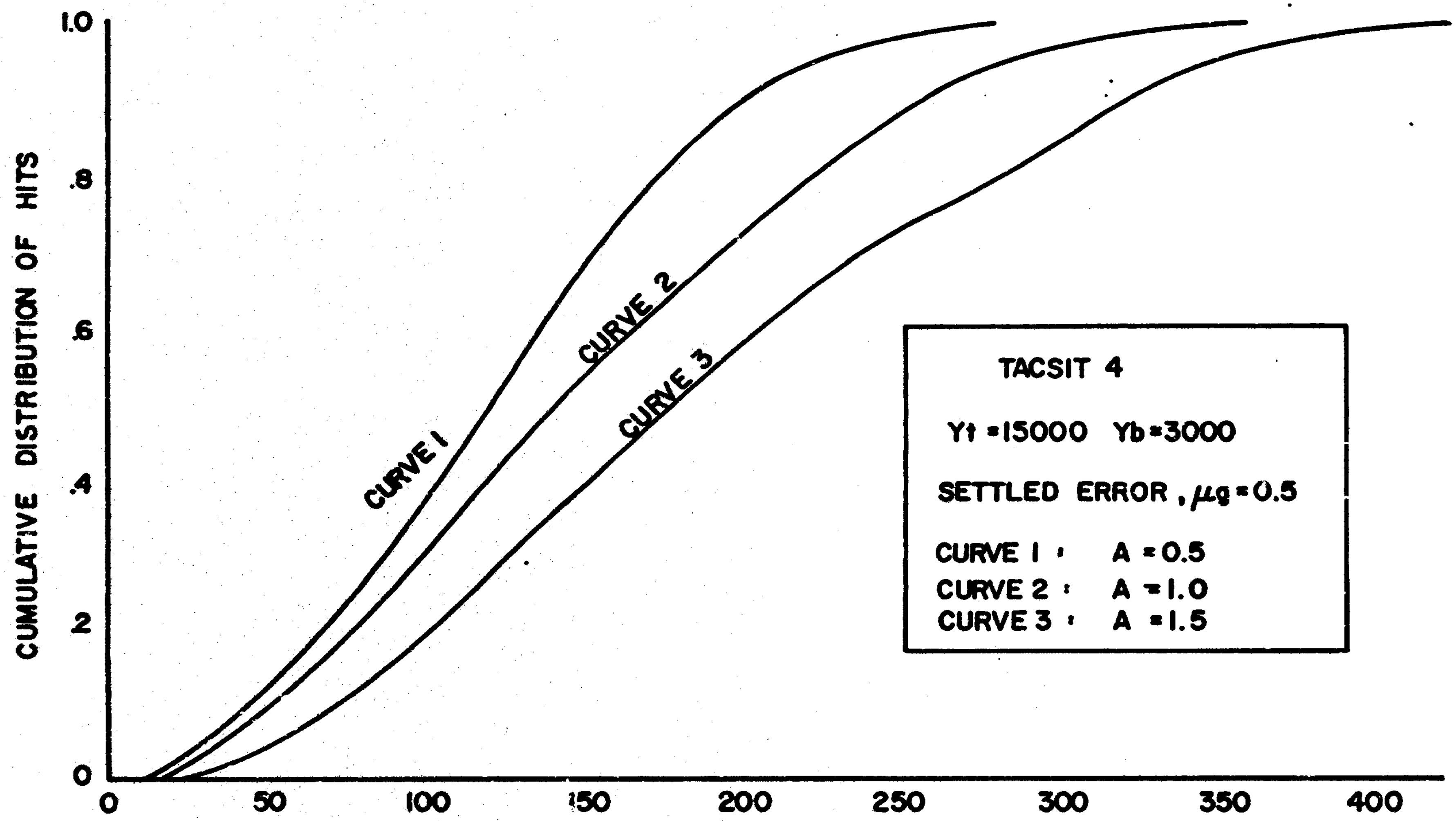

1.0
.8
.6
.4
.2
0
CUMULATIVE DISTRIBUTION OF HITS
0
50
100
150
200
250
300
350
400
RANGE FROM TARGET CENTER , YARDS
CURVE 1
CURVE 2
CURVE 3
TACSIT 4
Yt = 15000 Yb = 3000
EXCURSION, A = 1.5
CURVE 1 : μg = 0.0
CURVE 2 : μg = 0.5
CURVE 3 : μg = 1.0

FIG. 9


1.0
.8
.6
.4
.2
0
CUMULATIVE DISTRIBUTION OF HITS
CURVE 1
CURVE 2
CURVE 3
TACSIT 4
Yt =15000 Yb=3000
SETTLED ERROR , μg =0.0
CURVE 1 : A =0.5
CURVE 2 : A =1.0
CURVE 3 : A =1.5
0
50
100
150
200
250
300
350
400
RANGE FROM TARGET CENTER , YARDS
FIG. 10



TACSIT 4
Yt =15000 Yb =3000
SETTLED ERROR , μg = 0.5
CURVE 1 : A = 0.5
CURVE 2 : A = 1.0
CURVE 3 : A = 1.5
CURVE 1
CURVE 2
CURVE 3
CUMULATIVE DISTRIBUTION OF HITS
1.0
.8
.6
.4
.2
0
0
50
100
150
200
250
300
350
400
RANGE FROM TARGET CENTER , YARDS

FIG. 11

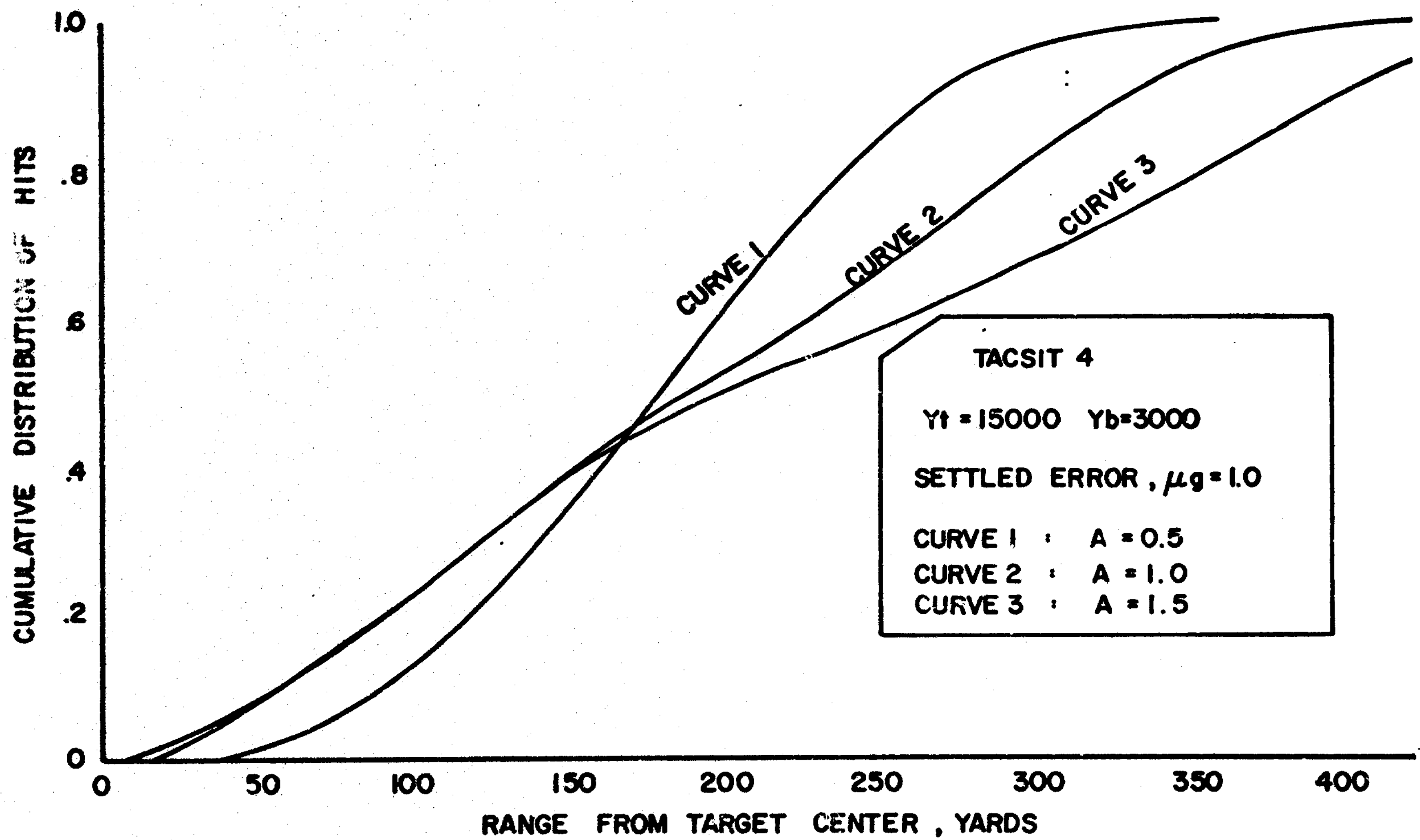
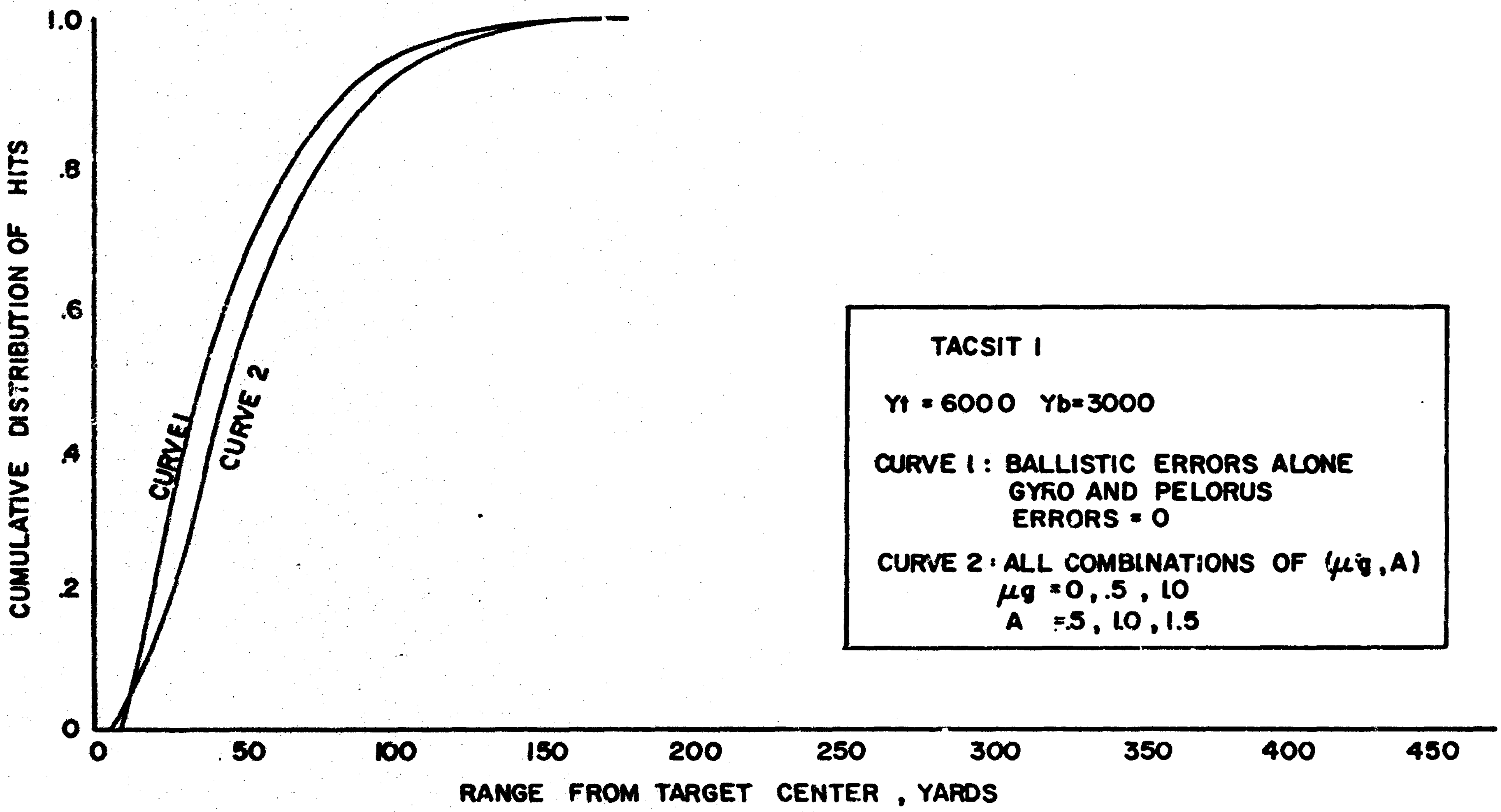
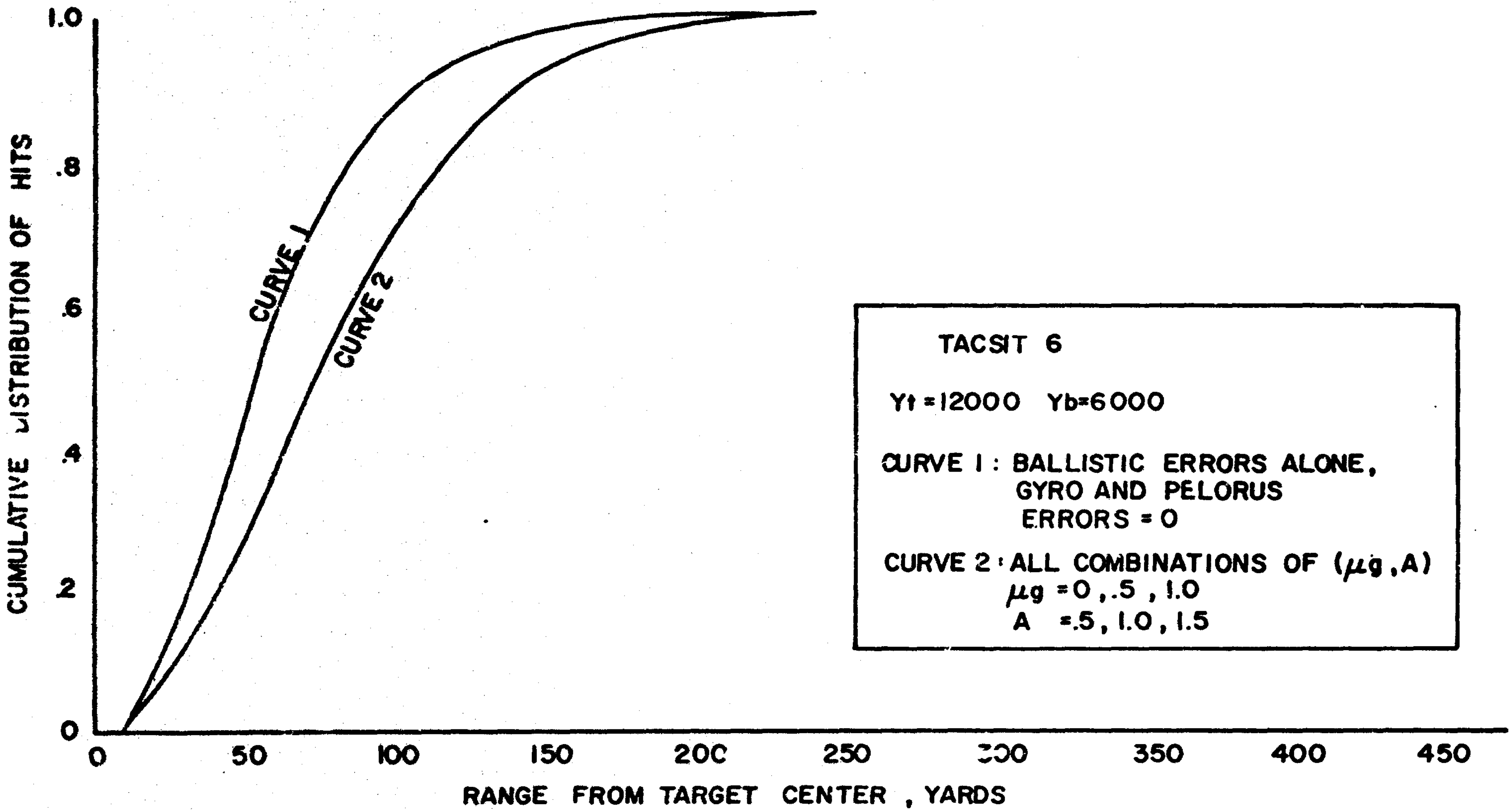

1.0
.8
.6
.4
.2
0
CUMULATIVE DISTRIBUTION OF HITS
0
50
100
150
200
250
300
350
400
RANGE FROM TARGET CENTER, YARDS
CURVE 1
CURVE 2
CURVE 3
TACSIT 4
Yt = 15000 Yb=3000
SETTLED ERROR, μg = 1.0
CURVE 1 : A = 0.5
CURVE 2 : A = 1.0
CURVE 3 : A = 1.5

FIG. 12


CUMULATIVE DISTRIBUTION OF HITS
1.0
.8
.6
.4
.2
0
CURVE 1
CURVE 2
0
50
100
150
200
250
300
350
400
450
RANGE FROM TARGET CENTER , YARDS
TACSIT I
Yt = 6000 Yb=3000
CURVE 1 : BALLISTIC ERRORS ALONE
GYRO AND PELORUS
ERRORS = 0
CURVE 2 : ALL COMBINATIONS OF (μg , A)
μg = 0 , .5 , 1.0
A = .5 , 1.0 , 1.5
FIG. 13



CUMULATIVE DISTRIBUTION OF HITS
1.0
.8
.6
.4
.2
0
CURVE 1
CURVE 2
TACSIT 6
Yt = 12000 Yb = 6000
CURVE 1 : BALLISTIC ERRORS ALONE, GYRO AND PELORUS ERRORS = 0
CURVE 2 : ALL COMBINATIONS OF (μg, A)
μg = 0, .5, 1.0
A = .5, 1.0, 1.5
0
50
100
150
20C
250
300
350
400
450
RANGE FROM TARGET CENTER , YARDS

FIG. 14

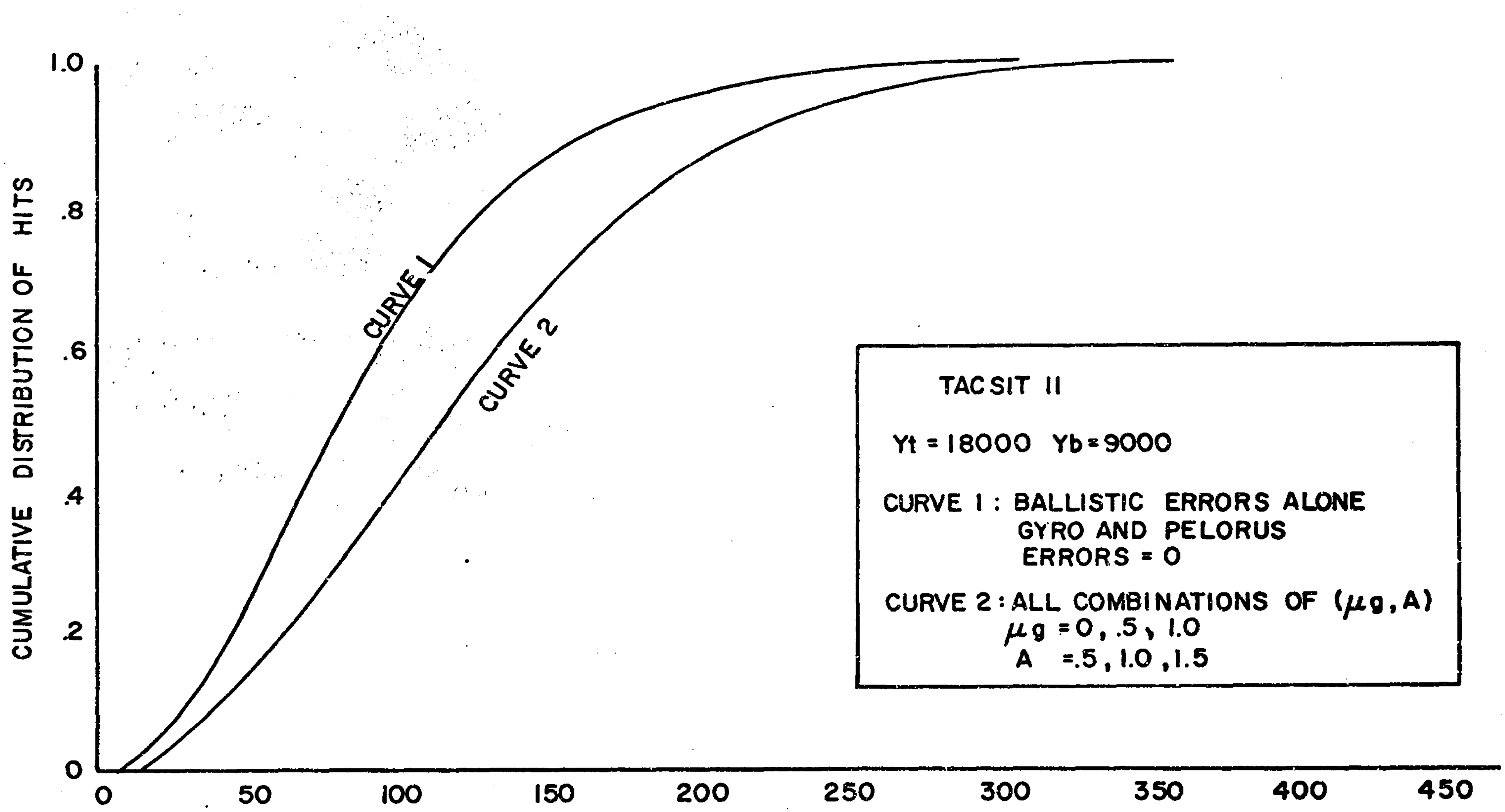

1.0
.8
.6
.4
.2
0
CUMULATIVE DISTRIBUTION OF HITS
0
50
100
150
200
250
300
350
400
450
RANGE FROM TARGET CENTER , YARDS
CURVE 1
CURVE 2
TAC SIT II
Yt = 18000 Yb = 9000
CURVE 1 : BALLISTIC ERRORS ALONE
GYRO AND PELORUS
ERRORS = 0
CURVE 2 : ALL COMBINATIONS OF (μg, A)
μg = 0, .5, 1.0
A = .5, 1.0, 1.5

FIG. 15

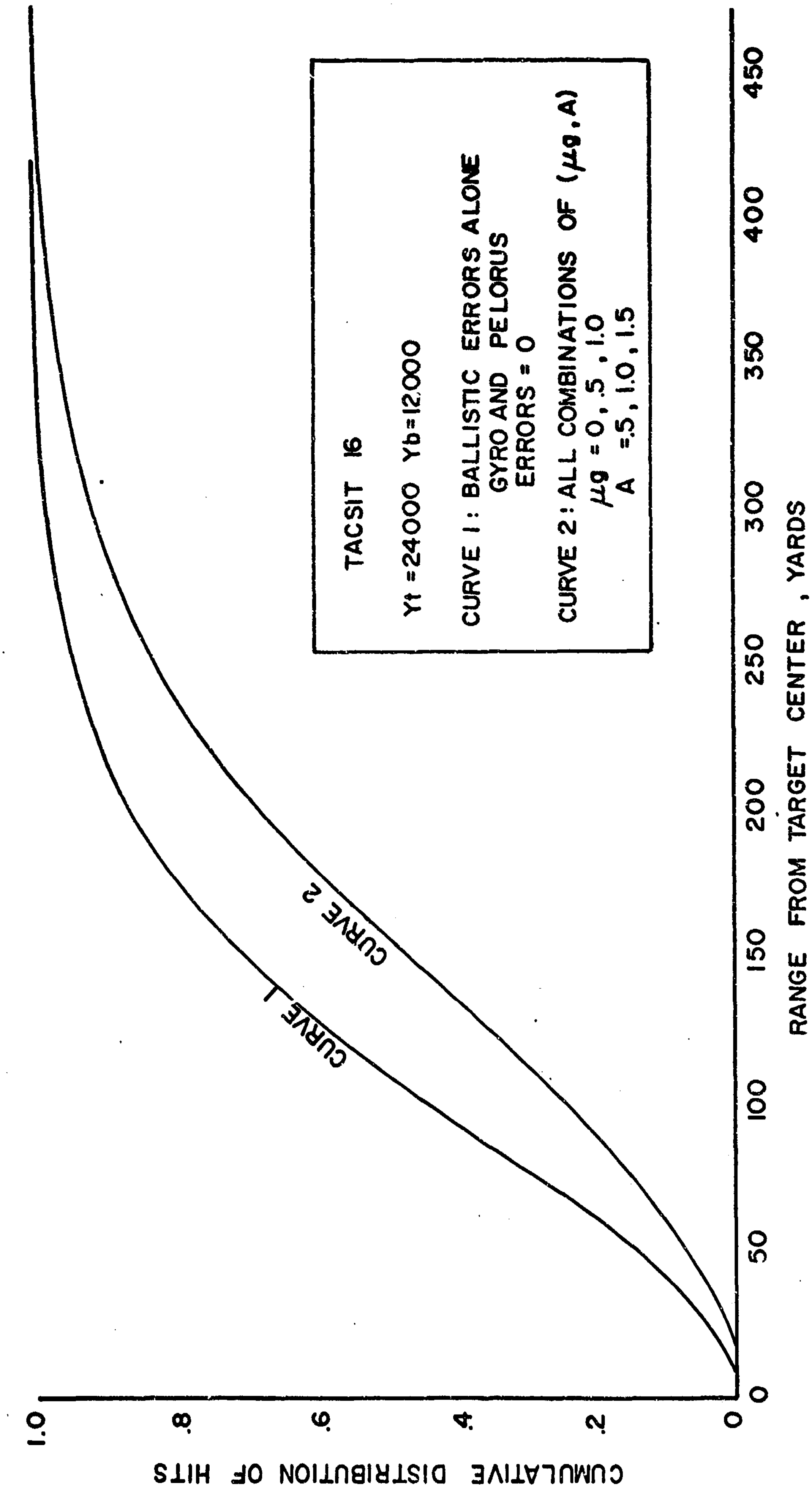

TACSIT 16
Yt =24000 Yb=12000
CURVE 1: BALLISTIC ERRORS ALONE
GYRO AND PELORUS
ERRORS = 0
CURVE 2: ALL COMBINATIONS OF (μg, A)
μg = 0, .5, 1.0
A = .5, 1.0, 1.5
CURVE 1
CURVE 2
CUMULATIVE DISTRIBUTION OF HITS
0
.2
.4
.6
.8
1.0
0
50
100
150
200
250
300
350
400
450
RANGE FROM TARGET CENTER , YARDS

FIG. 16

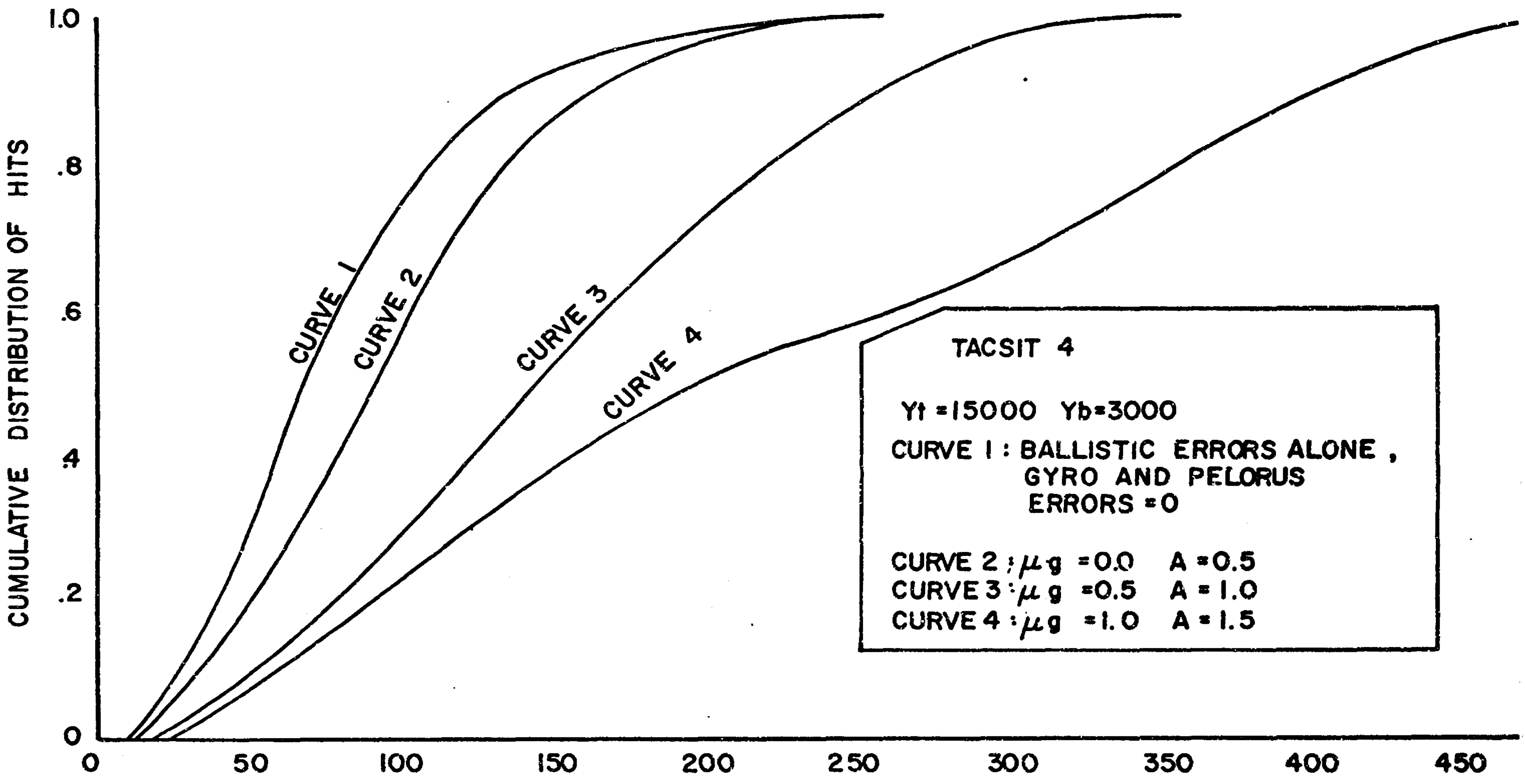
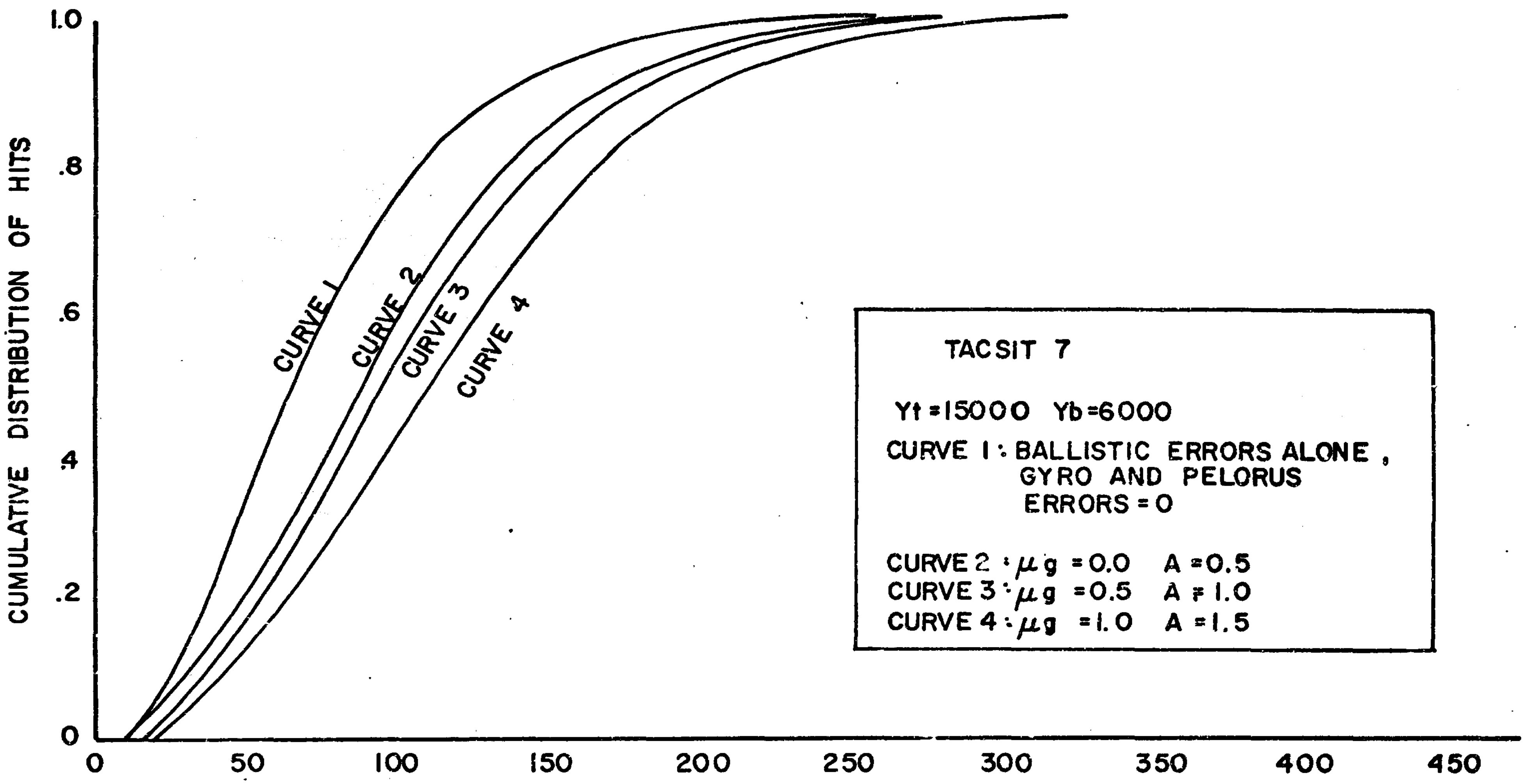
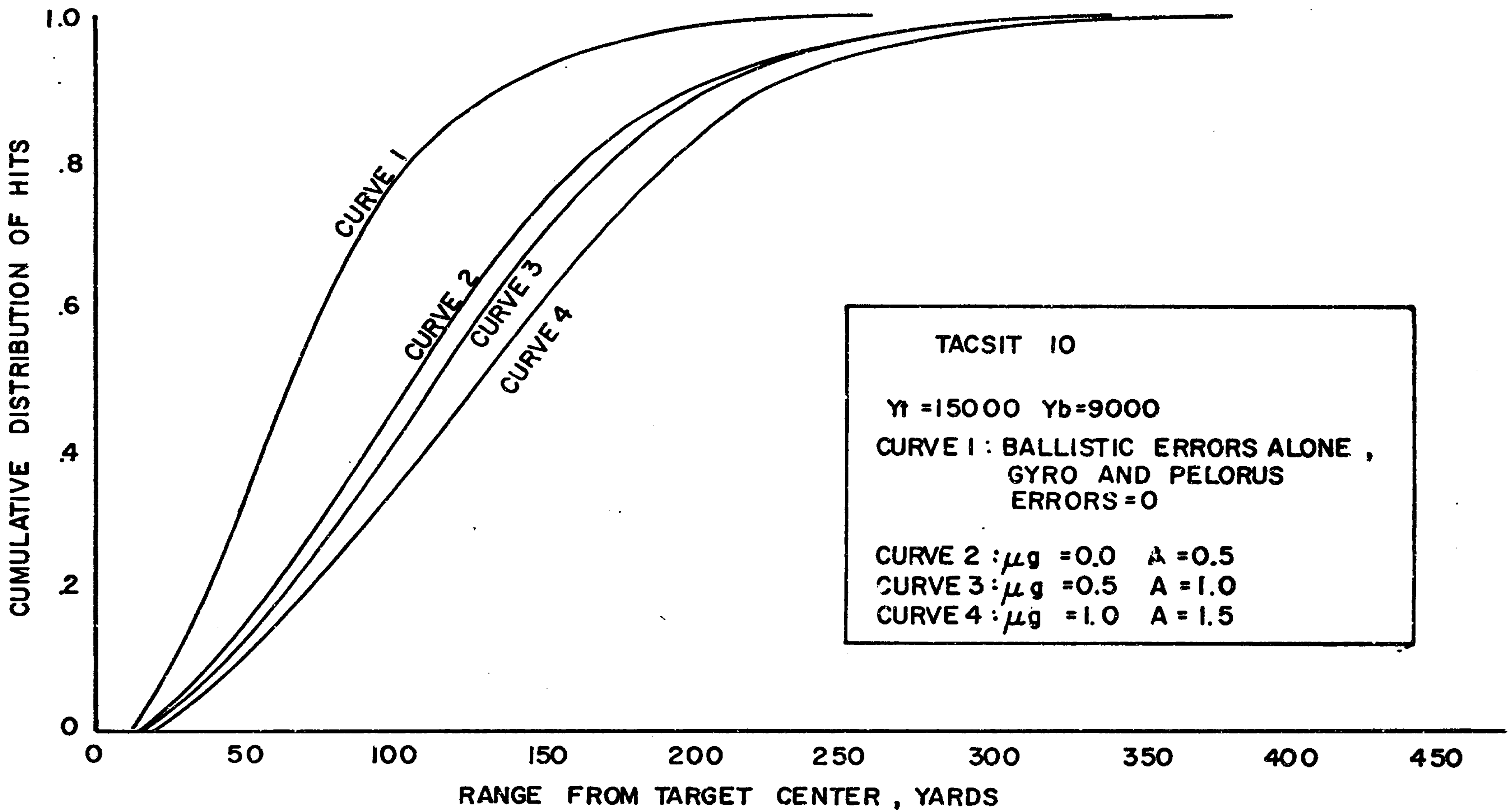

CUMULATIVE DISTRIBUTION OF HITS
1.0
.8
.6
.4
.2
0
0
50
100
150
200
250
300
350
400
450
RANGE FROM TARGET CENTER , YARDS
CURVE 1
CURVE 2
CURVE 3
CURVE 4
TACSIT 4
Yt =15000 Yb=3000
CURVE 1 : BALLISTIC ERRORS ALONE ,
GYRO AND PELORUS
ERRORS = 0
CURVE 2 : μg = 0.0 A = 0.5
CURVE 3 : μg = 0.5 A = 1.0
CURVE 4 : μg = 1.0 A = 1.5

FIG. 17



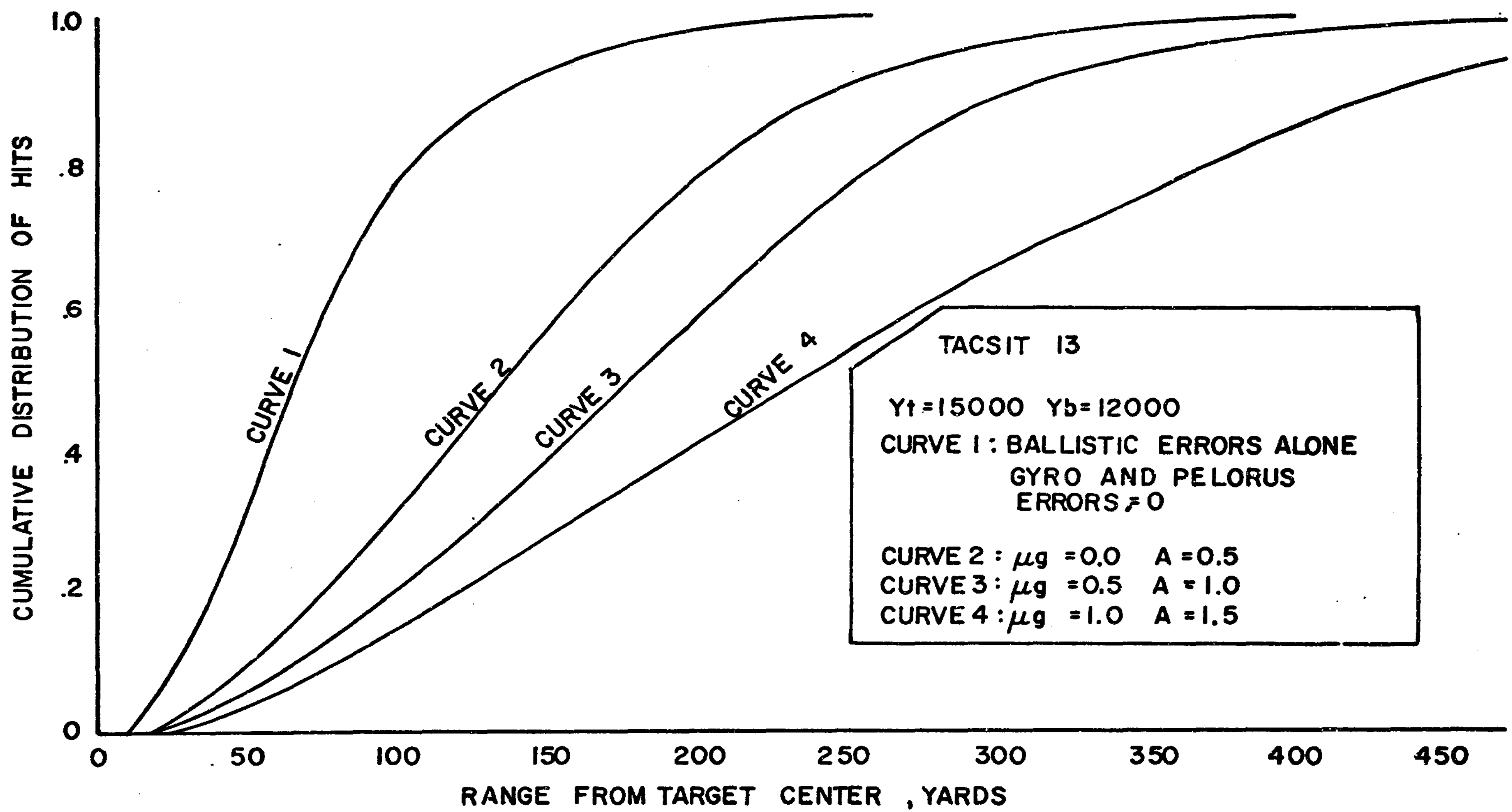
TACSIT 7
Yt =15000 Yb=6000
CURVE 1: BALLISTIC ERRORS ALONE, GYRO AND PELORUS ERRORS = 0
CURVE 2: μg = 0.0 A = 0.5
CURVE 3: μg = 0.5 A = 1.0
CURVE 4: μg = 1.0 A = 1.5
CURVE 1
CURVE 2
CURVE 3
CURVE 4
CUMULATIVE DISTRIBUTION OF HITS
RANGE FROM TARGET CENTER YARDS
0
.2
.4
.6
.8
1.0
0
50
100
150
200
250
300
350
400
450

FIG. 18



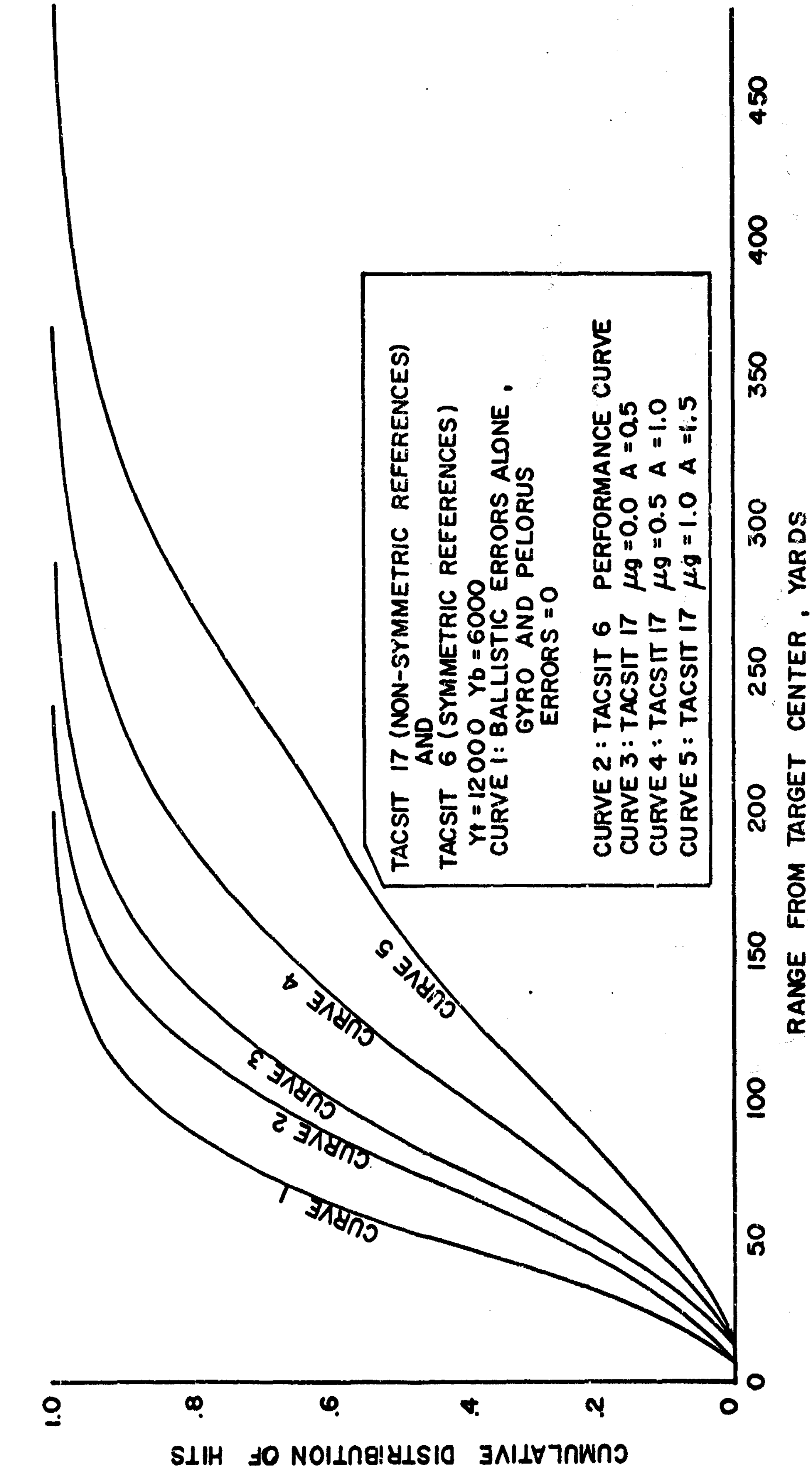
TACSIT 10
Yt =15000 Yb=9000
CURVE 1 : BALLISTIC ERRORS ALONE , GYRO AND PELORUS ERRORS=0
CURVE 2 : μg =0.0 A =0.5
CURVE 3 : μg =0.5 A =1.0
CURVE 4 : μg =1.0 A =1.5
CURVE 1
CURVE 2
CURVE 3
CURVE 4
CUMULATIVE DISTRIBUTION OF HITS
RANGE FROM TARGET CENTER , YARDS
0
.2
.4
.6
.8
1.0
0
50
100
150
200
250
300
350
400
450

FIG. 19



CUMULATIVE DISTRIBUTION OF HITS
1.0
.8
.6
.4
.2
0
0
50
100
150
200
250
300
350
400
450
RANGE FROM TARGET CENTER , YARDS
CURVE 1
CURVE 2
CURVE 3
CURVE 4
TACSIT 13
Yt=15000 Yb=12000
CURVE 1: BALLISTIC ERRORS ALONE
GYRO AND PELORUS
ERRORS ≠ 0
CURVE 2 : μg =0.0 A =0.5
CURVE 3 : μg =0.5 A = 1.0
CURVE 4 : μg =1.0 A =1.5

FIG. 20


TACSIT 17 (NON-SYMMETRIC REFERENCES)
AND
TACSIT 6 (SYMMETRIC REFERENCES)
Yt = 12000 Yb = 6000
CURVE 1: BALLISTIC ERRORS ALONE, GYRO AND PELORUS ERRORS = 0
CURVE 2 : TACSIT 6 PERFORMANCE CURVE
CURVE 3 : TACSIT 17 μg = 0.0 A = 0.5
CURVE 4 : TACSIT 17 μg = 0.5 A = 1.0
CURVE 5 : TACSIT 17 μg = 1.0 A = 1.5
CURVE 1
CURVE 2
CURVE 3
CURVE 4
CURVE 5
CUMULATIVE DISTRIBUTION OF HITS
0
.2
.4
.6
.8
1.0
0
50
100
150
200
250
300
350
400
450
RANGE FROM TARGET CENTER , YARDS
FIG. 21

VI. DISCUSSION OF RESULTS AND RECOMMENDATIONS

It was noted that in those cases where $|\mu_g| \geqq A$, there was a serious degradation in mission effectiveness against very small targets. Thus, the model suggests that the absolute value of settled error should not be allowed to exceed the maximum excursion about that settled error.

A detailed examination of the effects of gyro errors in tactical situation 4 revealed that variations in the gyro error parameters (μ_g, A) had a pronounced effect upon mission effectiveness. A comparison of the results observed in tactical situation 4 compared to those obtained in tactical situations 7, 10, and 13 ($Y_T = 15000$ yards in all four cases) indicated that the deleterious effect of the gyro error decreased as $Y_b \longrightarrow (Y_T - Y_b)$. It was further noted that in tactical situations 1, 6, 11, and 16 (where in each case $Y_b = (Y_T - Y_b)$) the model was insensitive to variations in gyro error. This indicates that by judicious use of information readily available, ship's personnel might be able to obviate the effects of gyro error in a shore bombardment mission.

The above noted conclusion is subject to two conditions. First, in tactical situations 1-16 the navigational references were symmetrically placed with respect to the ATL. A comparison of TACSIT 6 and 17 revealed that when this symmetry was lacking, the model became extremely sensitive to gyro error variations. It was hypothesized that in the symmetrical cases the errors in determining ship's position were predominantly along the X-axis, hence errors in range were small. On the other hand in the non-symmetric case, such as presented in TACSIT 17, a significant range error was introduced as a result of incorrectly determining ship's position.

The second condition, and one not specifically explored, involves time. The gyro error was assumed to be time dependent, and it was conjectured that the insensitivity to gyro error variations noted in TACSIT's 1, 6, 11, and 16 would not have been apparent if the model had not used an identical gyro error in concurrently determining ship's position and computing the bearing on which the guns were to be trained. It was therefore hypothesized that in order to obtain the results suggested by the model, ship's personnel would have to continually update the ship's navigational position in order that the same gyro error would be an input to both the navigational solution and the gun train solution.

In consideration of the observed results of the model, and the conditions mentioned, it appears that the effects of a ship's gyro error could be essentially nullified. The optimal trigonometric solution to arrive at the desired result would have to be determined in any particular tactical situation.

On method which might be employed in exploiting this result would involve predetermining the effects of various gyro errors over the range of errors anticipated for the tactical situation anticipated. Then by comparison of an accurately determined ship's position using all available means, with the (X'_0, Y'_0) components determined by pelorus bearings, the instantaneous gyro error could be determined, and an appropriate "offset" correction could be entered manually into the fire control computer. Additional corrections would have to be made at regular intervals to compensate for the time dependent changes in gyro error.

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Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP
3. REPORT TITLE The Effect of Sensor Accuracy on Naval Gunfire Support Mission Effectiveness	
4. DESCRIPTIVE NOTES (Type of report and, inclusive dates) Master's Thesis; September 1970	
5. AUTHOR(S) (First name, middle initial, last name) Thomas Clinton Winant	
6. REPORT DATE September 1970	7a. TOTAL NO. OF PAGES 56 7b. NO. OF REFS 7
8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
10. DISTRIBUTION STATEMENT This document has been approved for public release and sale; its distribution is unlimited	
11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

This thesis investigates the effects of errors in two shipboard sensors, the gyrocompass system and the peloruses, on a ship's mission effectiveness. The missions considered were a series of specially constrained shore bombardment missions. Various gyrocompass errors were investigated against area targets of varying radii.

The ultimate benefit which will hopefully be realized is that force commanders will be provided with a means to quantitatively evaluate the inherent capabilities of the various ships under their commands in assigning ships to specific missions.

In addition, a tactical innovation is suggested which could improve naval shore bombardment capabilities by partially countering the deleterious effect of ship's gyro error in indirect fire missions where spotting is not available.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Gyro error						
Pelorus error						
Sensor accuracy						
Mission effectiveness						
FORACS						
Shore bombardment						
Sensor accuracy						
Sensor error						
Naval gunfire support						

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S/N 0101-807-6821

END